

# Modeling Non-Consumptive Attributes in Choice Experiments

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Common feature in choice experiments.

Limited research / consensus on how to model them.

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Should / could  $\mathbf{z}$  have *nonzero marginal utility* even in the absence of  $\mathbf{x}$ ?

Could  $\mathbf{z}$  be related to unobserved heterogeneity?



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*Generalized choice situation  $G(X, d)$*

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*Generalized choice situation*  $G(X, d)$

$d =$  **ancillary condition**

- exogenous feature of choice environment
- not relevant to social planner's policy evaluation
- elements that change when choice is delegated
- BUT "can be analyst-specific"

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Pair  $(A, f) = \textit{Extended choice problem}$

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- labeling of “default alternative”

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Observable information that is *irrelevant in the rational assessment* of the alternatives

- orderings of choice options
- labeling of “default alternative”
- number of choice menus presented to individual

## Potential candidates in CEs / NMV

- Presentation of information (text, visual, etc.)

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- Presentation of information (text, visual, etc.)
- Wording of food labeling (“contains” / “does not contain”)
- Payment vehicle (tax, utility bill, etc.)
- Policy process / implementation (NGO, Gov. agency, etc.)
- Economic spillovers (secondary benefits to others)

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- potentially relevant for choice
- observable to analyst

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- $\mathbf{z}$  can, in theory, affect utility irrespective of  $\mathbf{x}$
- maximum econometric flexibility

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*I.J. Bateman et al. / Journal of Environmental Economics and Management 58 (2009) 106–118*

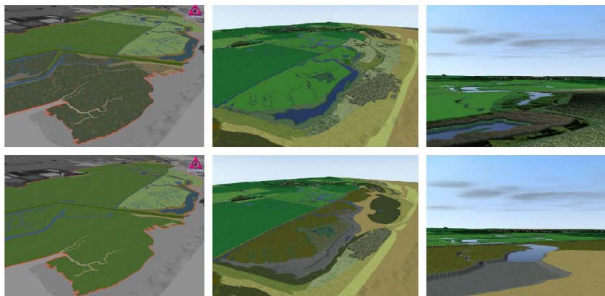


Fig. 1. VR visualisations of the status quo (upper row) and various alternative land use scenarios (lower row) from different viewing points.

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**Table 2. Attributes and Levels for Choice Experiment Design**

Attribute	Levels
Acres (4 levels)	<ol style="list-style-type: none"> <li>1. 20</li> <li>2. 60</li> <li>3. 100</li> <li>4. 200</li> </ol>
Land type (5 levels)	<ol style="list-style-type: none"> <li>1. Active farmland               <ol style="list-style-type: none"> <li>a. Nursery</li> <li>b. Food crop</li> <li>c. Dairy or livestock</li> </ol> </li> <li>2. Farmland (currently idle)</li> <li>3. Forest</li> </ol>
Policy technique and implementing agency (5 levels)	<ol style="list-style-type: none"> <li>1. Preservation contracts               <ol style="list-style-type: none"> <li>a. By state</li> <li>b. By land trusts using block grants</li> </ol> </li> <li>2. Outright purchase               <ol style="list-style-type: none"> <li>a. By state</li> <li>b. By land trusts using block grants</li> </ol> </li> <li>3. Conservation zoning</li> </ol>
Public access (3 levels)	<ol style="list-style-type: none"> <li>1. No access allowed</li> <li>2. Access for walking and biking</li> <li>3. Access for hunting</li> </ol>
Development risk (3 levels)	<ol style="list-style-type: none"> <li>1. Development likely in less than 10 years if not preserved</li> <li>2. Development likely in 10–30 years if not preserved</li> <li>3. Development NOT likely within the next 30 years</li> </ol>
Cost (6 levels)	<ol style="list-style-type: none"> <li>1. \$5</li> <li>2. \$15</li> <li>3. \$30</li> <li>4. \$50</li> <li>5. \$100</li> <li>6. \$200</li> </ol>

# Hu et al., 2005

$x$  = flour type, brand (local, national), GMO content

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Table 1. Levels within each attribute of pre-packaged sliced bread

	Level 1	Level 2	Level 3	Level 4
Brand name	Store brand	National brand	—	—
Type of flour	White	Partial (60%) whole wheat	Whole wheat (100%)	Multigrain
Price (CND)	\$0.99	\$1.49	\$2.49	\$3.49
GM or not	GM ingredients present	GM ingredients absent	Not specified (as in the mixed labelling scenario)	—

## Related Literature

Growing interest in correct model specification for CEs



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- Torres et al. (2011): Different functional forms for IUF

# Basic Setup

$$U_{i0}^* = \mathbf{x}'_{i0} \boldsymbol{\beta}_i^* + \gamma_i^* m_i + \epsilon_{i0}^*$$

$$U_{i1t}^* = \mathbf{x}'_{i1t} \boldsymbol{\beta}_i^* + \gamma_i^* (m_i + P_i) + \epsilon_{i1t}^*$$

# Model in preference-space

$$U_{it}^* = U_{i1t}^* - U_{i0}^* = \mathbf{x}'_{it}\boldsymbol{\beta}_i^* + \gamma_i^* P_i + \epsilon_{it}^* \quad \text{where}$$

$$\mathbf{x}_{it} = (\mathbf{x}_{i1t} - \mathbf{x}_{i0}) \quad \text{and} \quad \epsilon_{it}^* = (\epsilon_{i1t}^* - \epsilon_{i0}^*)$$

$$\epsilon_{it}^* \sim n(0, \sigma^2)$$

## Model in WTP-space

$$U_{it} = \frac{U_{i1t}^*}{\gamma_i^*} = \mathbf{x}'_{it}\beta_i + P_i + \epsilon_{it} \quad \text{where}$$

$$\beta_i = \frac{\beta_i^*}{\gamma_i^*} \quad \text{and} \quad \epsilon_{it} = \frac{\epsilon_{it}^*}{\gamma_i^*} \quad \text{with}$$

$$\epsilon_{it} \sim n(0, \sigma_i^2) \quad \text{and} \quad \sigma_i^2 = \frac{\sigma^2}{\gamma_i^{*2}}$$

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- Direct priors for attribute-specific WTP (“*implicit prices*”)
- Gain in accuracy and robustness in hierarchical models (Sonnier et al., 2007)
- More intuitive interpretation of individual heterogeneity

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- Assume unobserved heterogeneity in preferences throughout
- $\mathbf{z}_i$  is **respondent-specific** (invariant w/in  $i$ )
- Must be linked to  $\mathbf{x}_{it}$  (else drops out)

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- $z_i$  affects both **expectation** and **variance** of  $\beta_i$ 
  - Via shift / scaling

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$$V(\beta_i) = \Sigma$$

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$$V(\beta_{i,k}) = \Sigma_{kk}$$

Coefficient / expectation ratio

$$\frac{\beta_{i,k}}{E(\beta_{i,k})} = \frac{\beta_k + \delta_{i,k}}{\beta_k} = 1 + \frac{\delta_{i,k}}{\beta_k}$$

## Model S1a

$$\beta_i = \beta + \mathbf{A}z_i + \delta_i, \quad \text{where}$$

$$\mathbf{A} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1l} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{k1} & \alpha_{k2} & \dots & \alpha_{kl} \end{pmatrix}$$

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$$E(\beta_i) = \beta + \mathbf{A}z_i$$

$$V(\beta_i) = \Sigma$$

$$E(\beta_{i,k}) = \beta_k + \mathbf{z}'_i \alpha_k$$

$$V(\beta_{i,k}) = \Sigma_{kk}$$

# Model S1a, cont'd

Differential effect of binary CRNC

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Differential effect of binary CRNC

$$\begin{aligned} & \beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \\ & \beta_k + \alpha_{kl} + \delta_{i,k} - (\beta_k + \delta_{i,k}) = \alpha_{kl}, \forall i \end{aligned}$$

## Model S1a, cont'd

Differential effect of binary CRNC

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Coefficient / expectation ratios

$$\begin{aligned} \frac{\beta_{i,k} | (z_{i,l} = 0)}{E(\beta_{i,k} | (z_{i,l} = 0))} &= \frac{\beta_k + \delta_{i,k}}{\beta_k} = 1 + \frac{\delta_{i,k}}{\beta_k} \\ \frac{\beta_{i,k} | (z_{i,l} = 1)}{E(\beta_{i,k} | (z_{i,k} = 1))} &= \frac{\beta_k + \alpha_{kl} + \delta_{i,k}}{\beta_k + \alpha_{kl}} = 1 + \frac{\delta_{i,k}}{\beta_k + \alpha_{kl}} \end{aligned} \quad (1)$$



## Model S1b

$$\beta_i = (\mathbf{I} + \mathbf{\Lambda}) \beta + \delta_i, \quad \text{where}$$

$$\mathbf{\Lambda} = \begin{pmatrix} \mathbf{z}'_i \alpha_1 & 0 & \dots & 0 \\ 0 & \mathbf{z}'_i \alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{z}'_i \alpha_k \end{pmatrix},$$

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$$E(\beta_i) = (\mathbf{I} + \mathbf{\Lambda}) \beta$$

$$V(\beta_i) = \boldsymbol{\Sigma}$$

$$E(\beta_{i,k}) = (1 + \mathbf{z}'_i \boldsymbol{\alpha}_k) \beta_k$$

$$V(\beta_{i,k}) = \Sigma_{kk}$$

# Model S1b, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$(1 + \alpha_{kl}) \beta_k + \delta_{i,k} - (\beta_k + \delta_{i,k}) = \alpha_{kl} \beta_k, \forall i$$

## Model S1b, cont'd

Differential effect of binary CRNC

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Coefficient / expectation ratios

$$\begin{aligned} \frac{\beta_{i,k} | z_{i,l} = 0}{E(\beta_{i,k} | z_{i,l} = 0)} &= \frac{\beta_k + \delta_{i,k}}{\beta_k} = 1 + \frac{\delta_{i,k}}{\beta_k} \\ \frac{\beta_{i,k} | z_{i,l} = 1}{E(\beta_{i,k} | z_{i,k} = 1)} &= \frac{(1 + \alpha_{kl}) \beta_k + \delta_{i,k}}{(1 + \alpha_{kl}) \beta_k} = 1 + \frac{\delta_{i,k}}{(1 + \alpha_{kl}) \beta_k} \end{aligned}$$

## Model S2a

$$\beta_i = \beta + \Gamma \delta_i, \quad \text{where}$$

$$\Gamma = \begin{pmatrix} \mathbf{z}'_i \gamma_1 & 0 & \dots & 0 \\ 0 & \mathbf{z}'_i \gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{z}'_i \gamma_k \end{pmatrix},$$

$$\gamma_k = \{\gamma_{k1} \quad \gamma_{k2} \quad \dots \quad \gamma_{kl}\}',$$

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$$E(\beta_i) = \beta$$

$$V(\beta_i) = \Gamma \Sigma \Gamma'$$

$$E(\beta_{i,k}) = \beta_k$$

$$V(\beta_{i,k}) = (\mathbf{z}'_i \gamma_k)^2 \Sigma_{kk}$$

## Model S2a, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \\ \beta_k + \gamma_{kl} \delta_{i,k} - \beta_k = \gamma_{kl} \delta_{i,k}$$

## Model S2a, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \beta_k + \gamma_{kl}\delta_{i,k} - \beta_k = \gamma_{kl}\delta_{i,k}$$

Coefficient / expectation ratios

$$\frac{\beta_{i,k} | z_{i,l} = 0}{E(\beta_{i,k} | z_{i,l} = 0)} = \frac{\beta_k}{\beta_k} = 1$$

$$\frac{\beta_{i,k} | z_{i,l} = 1}{E(\beta_{i,k} | z_{i,l} = 1)} = 1 + \frac{\gamma_{kl}\delta_{i,k}}{\beta_k}$$



# Model S2b

$$\beta_i = \beta + (\mathbf{I} + \mathbf{\Gamma}) \delta_i$$

## Model S2b

$$\beta_i = \beta + (\mathbf{I} + \mathbf{\Gamma}) \delta_i$$

$$E(\beta_i) = \beta$$

$$V(\beta_i) = (\mathbf{I} + \mathbf{\Gamma}) \mathbf{\Sigma} (\mathbf{I} + \mathbf{\Gamma})'$$

$$E(\beta_{i,k}) = \beta_k$$

$$V(\beta_{i,k}) = (1 + \mathbf{z}'_i \boldsymbol{\gamma}_k)^2 \Sigma_{kk}$$

## Model S2b, cont'd

Differential effect of binary CRNC

$$\begin{aligned} & \beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \\ & \beta_k + (1 + \gamma_{kl}) \delta_{i,k} - \beta_k = (1 + \gamma_{kl}) \delta_{i,k} \end{aligned}$$

## Model S2b, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$\beta_k + (1 + \gamma_{kl}) \delta_{i,k} - \beta_k = (1 + \gamma_{kl}) \delta_{i,k}$$

Coefficient / expectation ratios

$$\frac{\beta_{i,k} | z_{i,l} = 0}{E(\beta_{i,k} | z_{i,l} = 0)} = \frac{\beta_k}{\beta_k} = 1$$

$$\frac{\beta_{i,k} | z_{i,l} = 1}{E(\beta_{i,k} | z_{i,l} = 1)} = 1 + \frac{(1 + \gamma_{kl}) \delta_{i,k}}{\beta_k}$$

# Model S3aa

$$\beta_i = \beta + \mathbf{A}z_i + \mathbf{\Gamma}\delta_i$$

# Model S3aa

$$\beta_i = \beta + \mathbf{A}z_i + \Gamma\delta_i$$

$$E(\beta_i) = \beta + \mathbf{A}z_i$$

$$V(\beta_i) = \Gamma\Sigma\Gamma'$$

$$E(\beta_{i,k}) = \beta_k + \mathbf{z}'_i\alpha_k$$

$$V(\beta_{i,k}) = (\mathbf{z}'_i\gamma_k)^2 \Sigma_{kk}$$

## Model S3aa, cont'd

Differential effect of binary CRNC

$$\begin{aligned} & \beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \\ & \beta_k + \alpha_{kl} + \gamma_{kl} \delta_{i,k} - \beta_k = \alpha_{kl} + \gamma_{kl} \delta_{i,k} \end{aligned}$$

## Model S3aa, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$\beta_k + \alpha_{kl} + \gamma_{kl}\delta_{i,k} - \beta_k = \alpha_{kl} + \gamma_{kl}\delta_{i,k}$$

Coefficient / expectation ratios

$$\frac{\beta_{i,k} | z_{i,l} = 0}{E(\beta_{i,k} | z_{i,l} = 0)} = \frac{\beta_k}{\beta_k} = 1$$

$$\frac{\beta_{i,k} | z_{i,l} = 1}{E(\beta_{i,k} | z_{i,k} = 1)} = 1 + \frac{\gamma_{kl}\delta_{i,k}}{\beta_k + \alpha_{kl}}$$



# Model S3ba

$$\beta_i = (\mathbf{I} + \mathbf{\Lambda})\beta + \mathbf{\Gamma}\delta_i$$

# Model S3ba

$$\beta_i = (\mathbf{I} + \mathbf{\Lambda})\beta + \mathbf{\Gamma}\delta_i$$

$$E(\beta_i) = (\mathbf{I} + \mathbf{\Lambda})\beta$$

$$V(\beta_i) = \mathbf{\Gamma}\mathbf{\Sigma}\mathbf{\Gamma}'$$

$$E(\beta_{i,k}) = (1 + \mathbf{z}'_i\boldsymbol{\alpha}_k)\beta_k$$

$$V(\beta_{i,k}) = (\mathbf{z}'_i\boldsymbol{\gamma}_k)^2 \Sigma_{kk}$$

## Model S3ba, cont'd

Differential effect of binary CRNC

$$\begin{aligned} & \beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \\ & (1 + \alpha_{kl}) \beta_k + \gamma_{kl} \delta_{i,k} - \beta_k = \alpha_{kl} \beta_k + \gamma_{kl} \delta_{i,k} \end{aligned}$$

## Model S3ba, cont'd

Differential effect of binary CRNC

$$\begin{aligned} \beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) = \\ (1 + \alpha_{kl}) \beta_k + \gamma_{kl} \delta_{i,k} - \beta_k = \alpha_{kl} \beta_k + \gamma_{kl} \delta_{i,k} \end{aligned}$$

Coefficient / expectation ratios

$$\begin{aligned} \frac{\beta_{i,k} | z_{i,l} = 0}{E(\beta_{i,k} | z_{i,l} = 0)} &= \frac{\beta_k}{\beta_k} = 1 \\ \frac{\beta_{i,k} | z_{i,l} = 1}{E(\beta_{i,k} | z_{i,l} = 1)} &= 1 + \frac{\gamma_{kl} \delta_{i,k}}{(1 + \alpha_{kl}) \beta_k} \end{aligned}$$

# Model S3ab

$$\beta_i = \beta + \mathbf{A}z_i + (\mathbf{I} + \mathbf{\Gamma}) \delta_i$$

# Model S3ab

$$\beta_i = \beta + \mathbf{A}z_i + (\mathbf{I} + \mathbf{\Gamma}) \delta_i$$

$$E(\beta_i) = \beta + \mathbf{A}z_i$$

$$V(\beta_i) = (\mathbf{I} + \mathbf{\Gamma}) \mathbf{\Sigma} (\mathbf{I} + \mathbf{\Gamma})'$$

$$E(\beta_{i,k}) = \beta_k + \mathbf{z}'_i \alpha_k$$

$$V(\beta_{i,k}) = (\mathbf{1} + \mathbf{z}'_i \gamma_k)^2 \Sigma_{kk}$$

# Model S3ab, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$\beta_k + \alpha_{kl} + (1 + \gamma_{kl}) \delta_{ik} - (\beta_k + \delta_{i,k}) = \alpha_{kl} + \gamma_{kl} \delta_{i,k}$$

## Model S3ab, cont'd

### Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$\beta_k + \alpha_{kl} + (1 + \gamma_{kl}) \delta_{ik} - (\beta_k + \delta_{i,k}) = \alpha_{kl} + \gamma_{kl} \delta_{i,k}$$

### Coefficient / expectation ratios

$$\frac{\beta_{i,k} | (z_{i,l} = 0)}{E(\beta_{i,k} | (z_{i,l} = 0))} = \frac{\beta_k + \delta_{i,k}}{\beta_k} = 1 + \frac{\delta_{ik}}{\beta_k}$$

$$\frac{\beta_{i,k} | (z_{i,l} = 1)}{E(\beta_{i,k} | (z_{i,k} = 1))} = \frac{(\beta_k + \alpha_{kl}) + (1 + \gamma_{kl}) \delta_{ik}}{\beta_k + \alpha_{kl}} = 1 + \frac{(1 + \gamma_{kl}) \delta_{ik}}{\beta_k + \alpha_{kl}}$$



# Model S3bb

$$\beta_i = (\mathbf{I} + \mathbf{\Lambda})\beta + (\mathbf{I} + \mathbf{\Gamma})\delta_i$$

# Model S3bb

$$\beta_i = (\mathbf{I} + \mathbf{\Lambda}) \beta + (\mathbf{I} + \mathbf{\Gamma}) \delta_i$$

$$E(\beta_i) = (\mathbf{I} + \mathbf{\Lambda}) \beta$$

$$V(\beta_i) = (\mathbf{I} + \mathbf{\Gamma}) \mathbf{\Sigma} (\mathbf{I} + \mathbf{\Gamma})'$$

$$E(\beta_{i,k}) = (1 + \mathbf{z}'_i \alpha_k) \beta_k$$

$$V(\beta_{i,k}) = (1 + \mathbf{z}'_i \gamma_k)^2 \Sigma_{kk}$$

# Model S3bb, cont'd

Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$(1 + \alpha_{kl}) \beta_k + (1 + \gamma_{kl}) \delta_{ik} - (\beta_k + \delta_{i,k}) = \alpha_{kl} \beta_k + \gamma_{kl} \delta_{i,k}$$

## Model S3bb, cont'd

### Differential effect of binary CRNC

$$\beta_{i,k} | (z_{i,l} = 1) - \beta_{i,k} | (z_{i,l} = 0) =$$

$$(1 + \alpha_{kl}) \beta_k + (1 + \gamma_{kl}) \delta_{ik} - (\beta_k + \delta_{i,k}) = \alpha_{kl} \beta_k + \gamma_{kl} \delta_{i,k}$$

### Coefficient / expectation ratios

$$\frac{\beta_{i,k} | (z_{i,l} = 0)}{E(\beta_{i,k} | (z_{i,l} = 0))} = \frac{\beta_k + \delta_{i,k}}{\beta_k} = 1 + \frac{\delta_{ik}}{\beta_k}$$

$$\frac{\beta_{i,k} | (z_{i,l} = 1)}{E(\beta_{i,k} | (z_{i,k} = 1))} = \frac{(1 + \alpha_{kl}) \beta_k + (1 + \gamma_{kl}) \delta_{ik}}{(1 + \alpha_{kl}) \beta_k} = 1 + \frac{(1 + \gamma_{kl}) \delta_{ik}}{(1 + \alpha_{kl}) \beta_k}$$

# Estimation

Hierarchical structure of model:

# Estimation

Hierarchical structure of model:

$$\beta_i \sim n(f(\beta, \mathbf{A}, \mathbf{z}_i), g(\boldsymbol{\Sigma}, \boldsymbol{\Gamma}, \mathbf{z}_i))$$

$$\sigma_i^2 \sim ig(\nu_0, \tau)$$

$$\beta \sim n(\boldsymbol{\mu}_{\beta,0}, \mathbf{V}_{\beta,0})$$

$$\mathbf{A} \sim n(\boldsymbol{\mu}_{\mathbf{A},0}, \mathbf{V}_{\mathbf{A},0})$$

$$\boldsymbol{\Gamma} \sim n(\boldsymbol{\mu}_{\boldsymbol{\Gamma},0}, \mathbf{V}_{\boldsymbol{\Gamma},0})$$

$$\tau \sim gam(\eta_0, \psi_0)$$

Joint posterior distribution (example models S3):

Joint posterior distribution (example models S3):

$$\begin{aligned}
 & p(\boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{A}, \boldsymbol{\Gamma}, \tau, \{\beta_i\}, \{\sigma_i^2\}, \mathbf{U} | \mathbf{y}, \mathbf{X}, \mathbf{Z}) \propto \\
 & p(\boldsymbol{\beta} | \boldsymbol{\Sigma}) p(\boldsymbol{\Sigma}) p(\mathbf{A}) p(\boldsymbol{\Gamma}) p(\tau) * \\
 & p(\{\beta_i\} | \boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{A}, \boldsymbol{\Gamma}, \mathbf{Z}) p(\{\sigma_i^2\} | \tau) * \\
 & p(\mathbf{U} | \{\beta_i\}, \{\sigma_i^2\}, \mathbf{X}) p(\mathbf{y} | \mathbf{U})
 \end{aligned}$$



Joint posterior distribution (example models S3):

$$\begin{aligned}
 & p(\boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{A}, \boldsymbol{\Gamma}, \tau, \{\beta_i\}, \{\sigma_i^2\}, \mathbf{U} | \mathbf{y}, \mathbf{X}, \mathbf{Z}) \propto \\
 & p(\boldsymbol{\beta} | \boldsymbol{\Sigma}) p(\boldsymbol{\Sigma}) p(\mathbf{A}) p(\boldsymbol{\Gamma}) p(\tau) * \\
 & p(\{\beta_i\} | \boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{A}, \boldsymbol{\Gamma}, \mathbf{Z}) p(\{\sigma_i^2\} | \tau) * \\
 & p(\mathbf{U} | \{\beta_i\}, \{\sigma_i^2\}, \mathbf{X}) p(\mathbf{y} | \mathbf{U})
 \end{aligned}$$

→ Gibbs Sampler

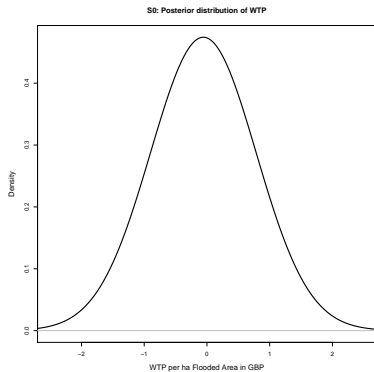
# Results

## Model Comparison

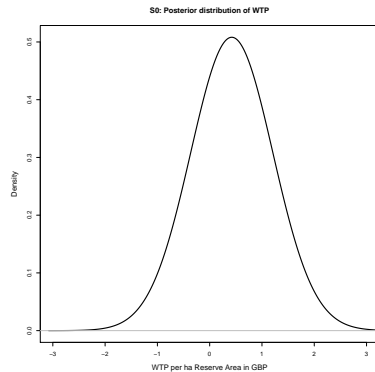
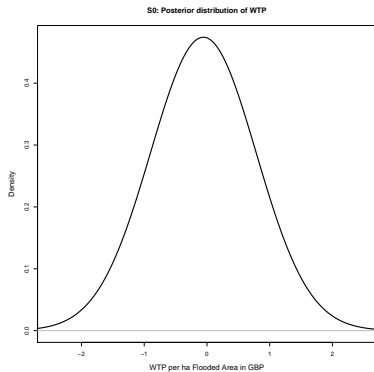
Model	MAD	Hit Rate
S0	0.263	0.757
S1a	0.228	0.790
S2b	0.220	0.783
S3a	0.216	0.786
S3b	0.226	0.778

# Model S0

# Model S0

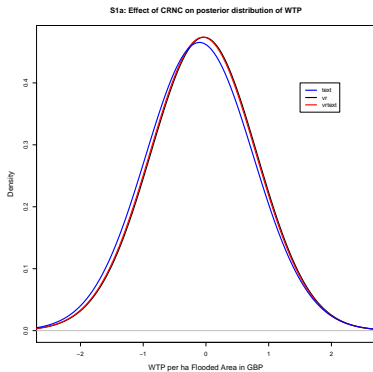


# Model S0



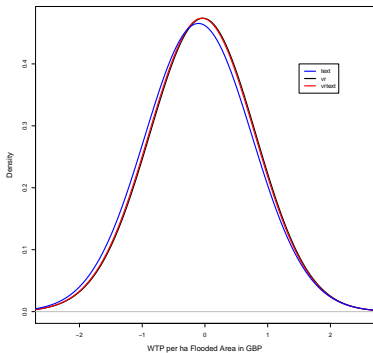
# Model S1a

# Model S1a

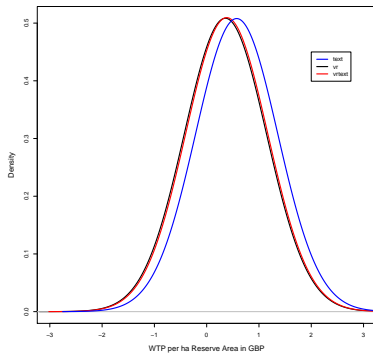


# Model S1a

S1a: Effect of CRNC on posterior distribution of WTP



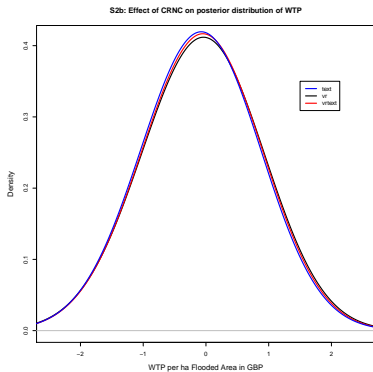
S1a: Effect of CRNC on posterior distribution of WTP



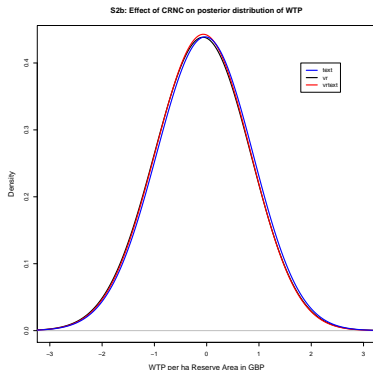
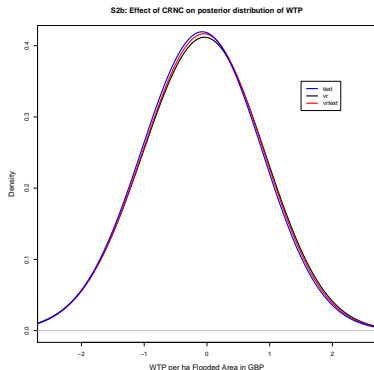


# Model S2b

# Model S2b

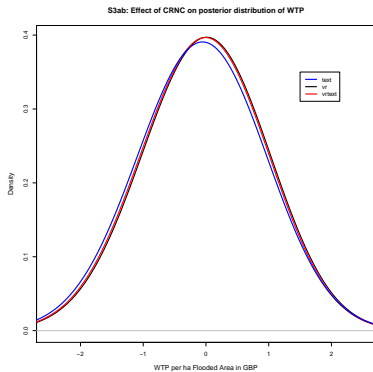


## Model S2b

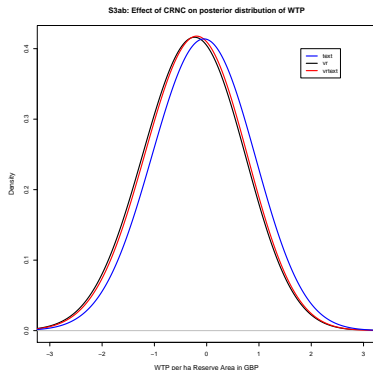
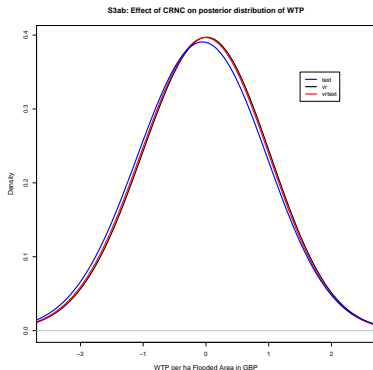


# Model S3ab

# Model S3ab

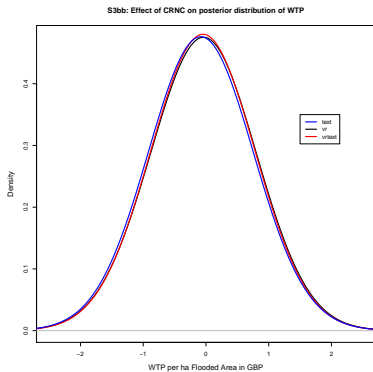


# Model S3ab



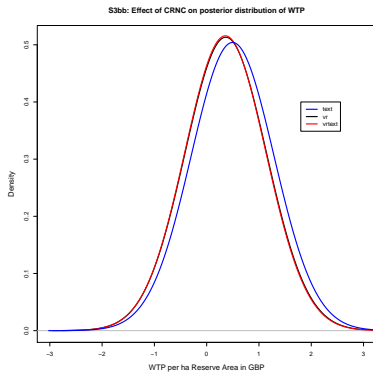
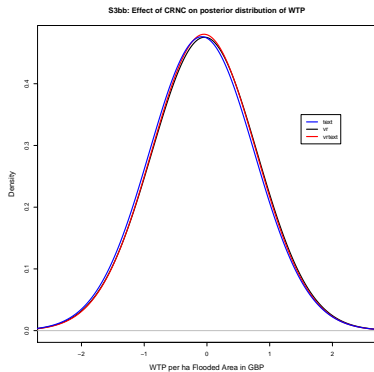
# Model S3bb

# Model S3bb



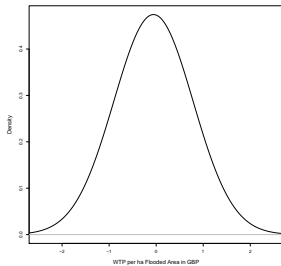


## Model S3bb

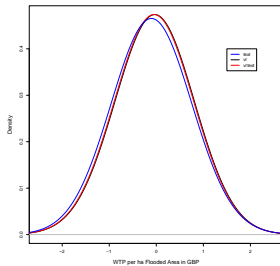


## flooded

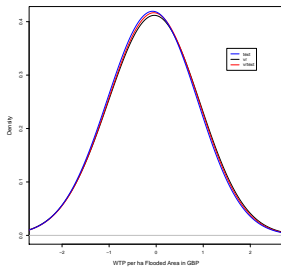
S0: Posterior distribution of WTP



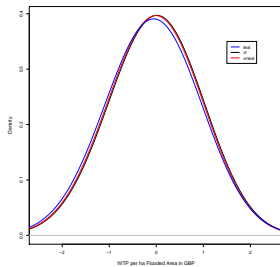
S1a: Effect of CRNC on posterior distribution of WTP



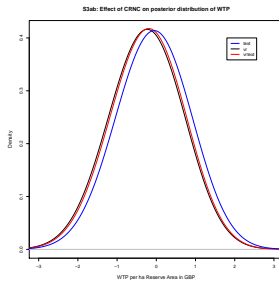
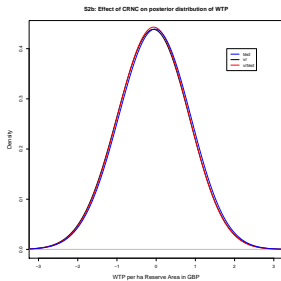
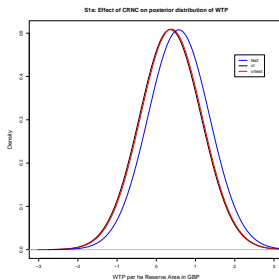
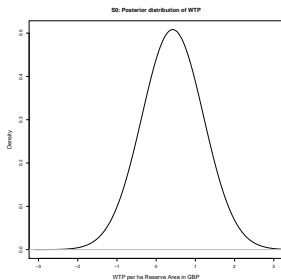
S2b: Effect of CRNC on posterior distribution of WTP



S3ab: Effect of CRNC on posterior distribution of WTP



## reserve



## Next steps

- Compute marginal likelihood and **posterior model probabilities**

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THANK YOU!