
Investigating Power Laws in
Hurricane Damages
Calvin Blackwell
College of Charleston
Special Thanks to Kathryn Williams

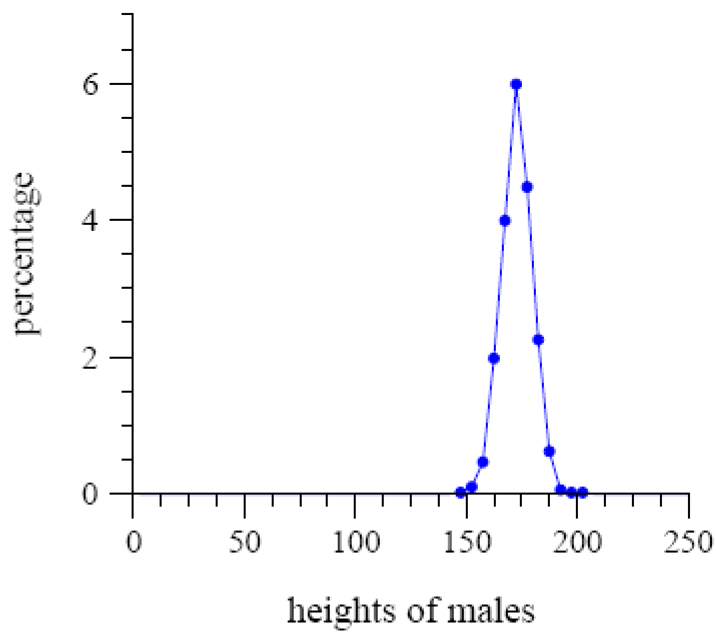
Paper Summary

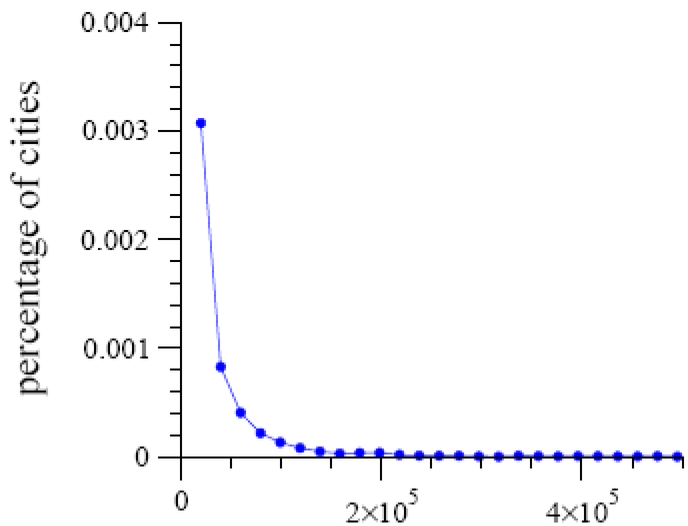
- The distribution of yearly damages due to hurricanes in the US is ‘fat-tailed.’
- The tail of the distribution is so ‘fat’ that the variance of damages, conditional on being in the tail, is potentially unbounded.

Most Damaging Hurricanes, 1900 – 2005 (Pielke et al. 2008)

Hurricane	**Year**	**Normalized Damages**
Greater Miami	1926	$157 B
Katrina	2005	$81 B
Galveston (1)	1900	$78 B
Galveston (2)	1915	$62 B
Andrew	1992	$58 B
New England	1938	$37 B

Power Law Distributions





Normal Distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

Power Law Distribution:

$$f(x) = \frac{\zeta - 2}{(x_{min})^{\zeta}} (x)^{-(\zeta+1)}$$

Discovered Power Laws

- City Size
- Asset Market Movements
- Earthquakes (Richter Scale is a logarithmic scale)
- Hurricanes (Corral et al. 2010)
- Lots of others

Tail Effects (Nordhaus, 2012)

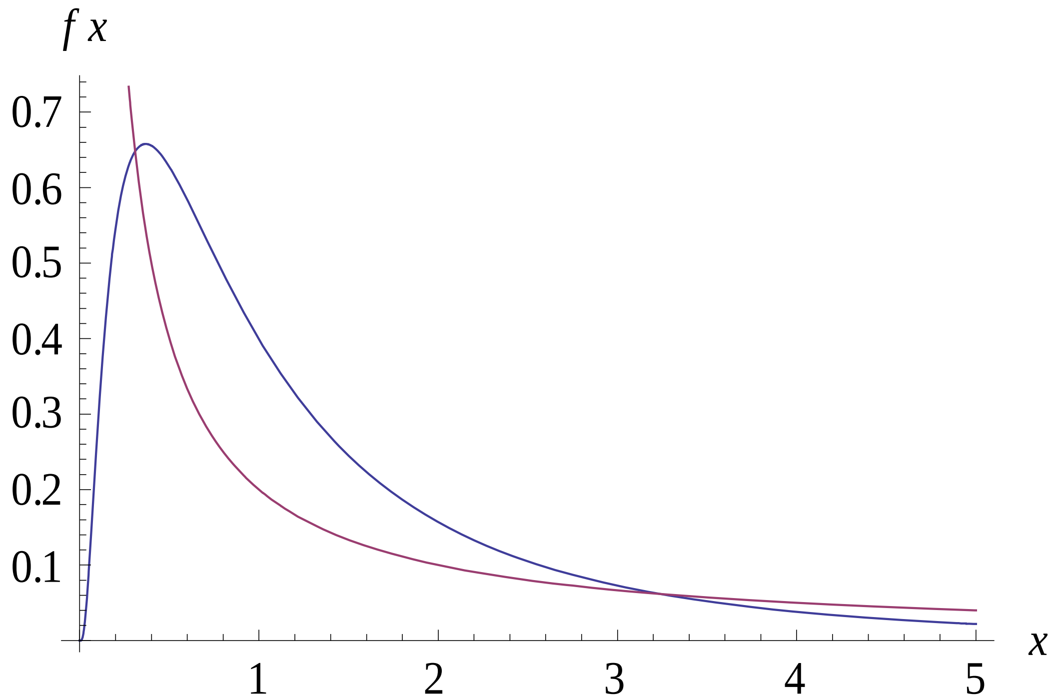
- For power law tails, conditional on being in the tail, the distribution of extreme events is wide.
  - Ex: Normal distribution 200 year event
    - Typical event is 12% larger than 200 year event
  - Ex: Power law distribution 200 year event
    - For earthquakes ($\zeta < 1$), typical event is 1000% larger than 200 year event
    - 2011 Japanese earthquake biggest ever in Japan!

Interesting Facts About Power Laws

- Two parameters: ζ and x_{min}
- ζ determines all the moments of the distribution.
  - If $\zeta < 1$, mean is divergent
  - If $\zeta < 2$, variance is divergent
  - pattern holds for higher moments
- Smaller ζ means 'fatter' tail
- x_{min} determines where power law behavior begins

Question

- Do hurricane damages in US have a power law tail?
- If so, what is ζ?
- Or is distribution
  - lognormal?
  - exponential?



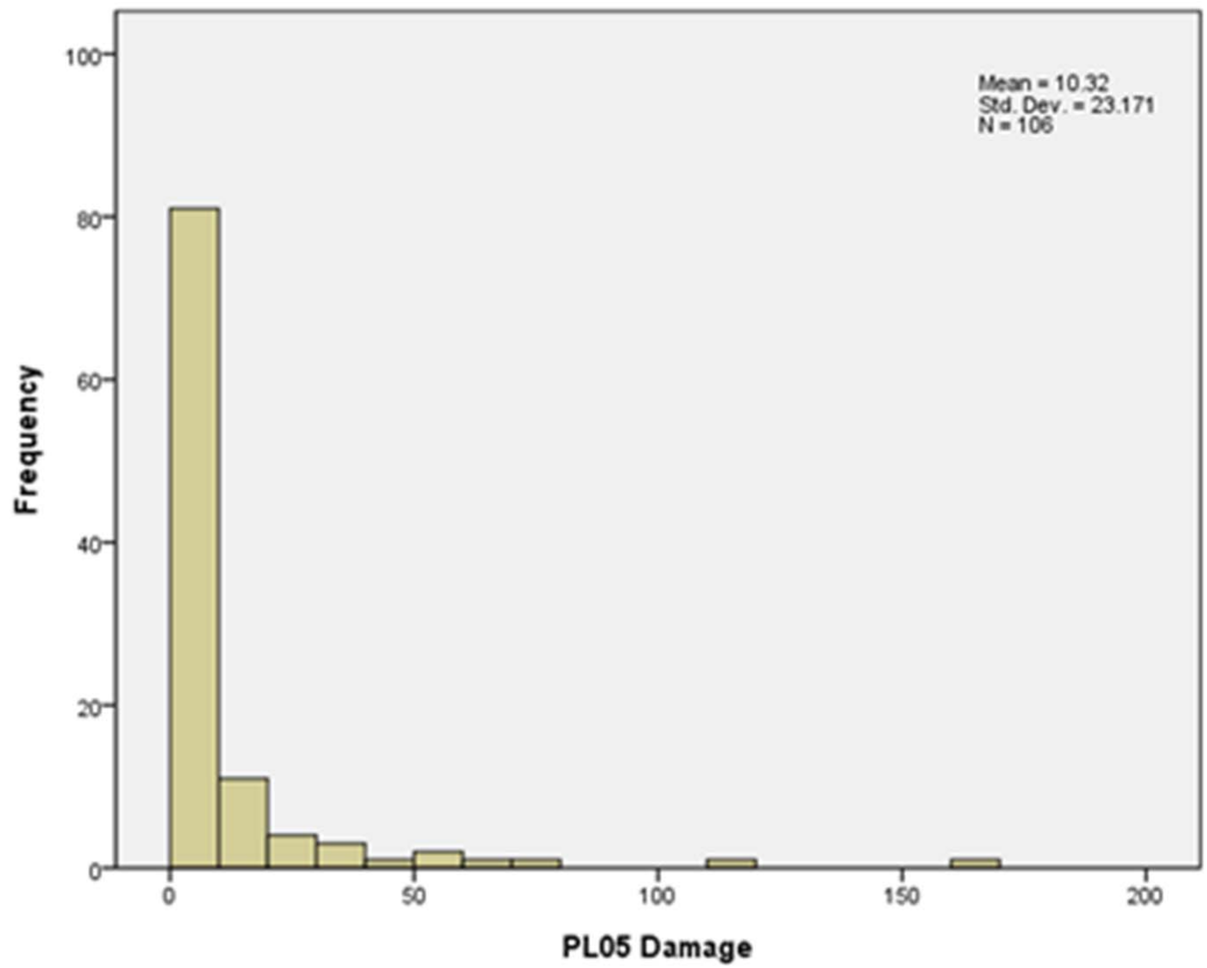
Data

- Pielke et al. (2008) data set of normalized economic hurricane damages from 1900 – 2005
- Damages = direct losses determined immediately after hurricane
- Damages normalized to be estimated as if the (historical) hurricane made landfall under contemporary levels of societal development
  - Controls for inflation, wealth and population
- Damages aggregated by year, not hurricane

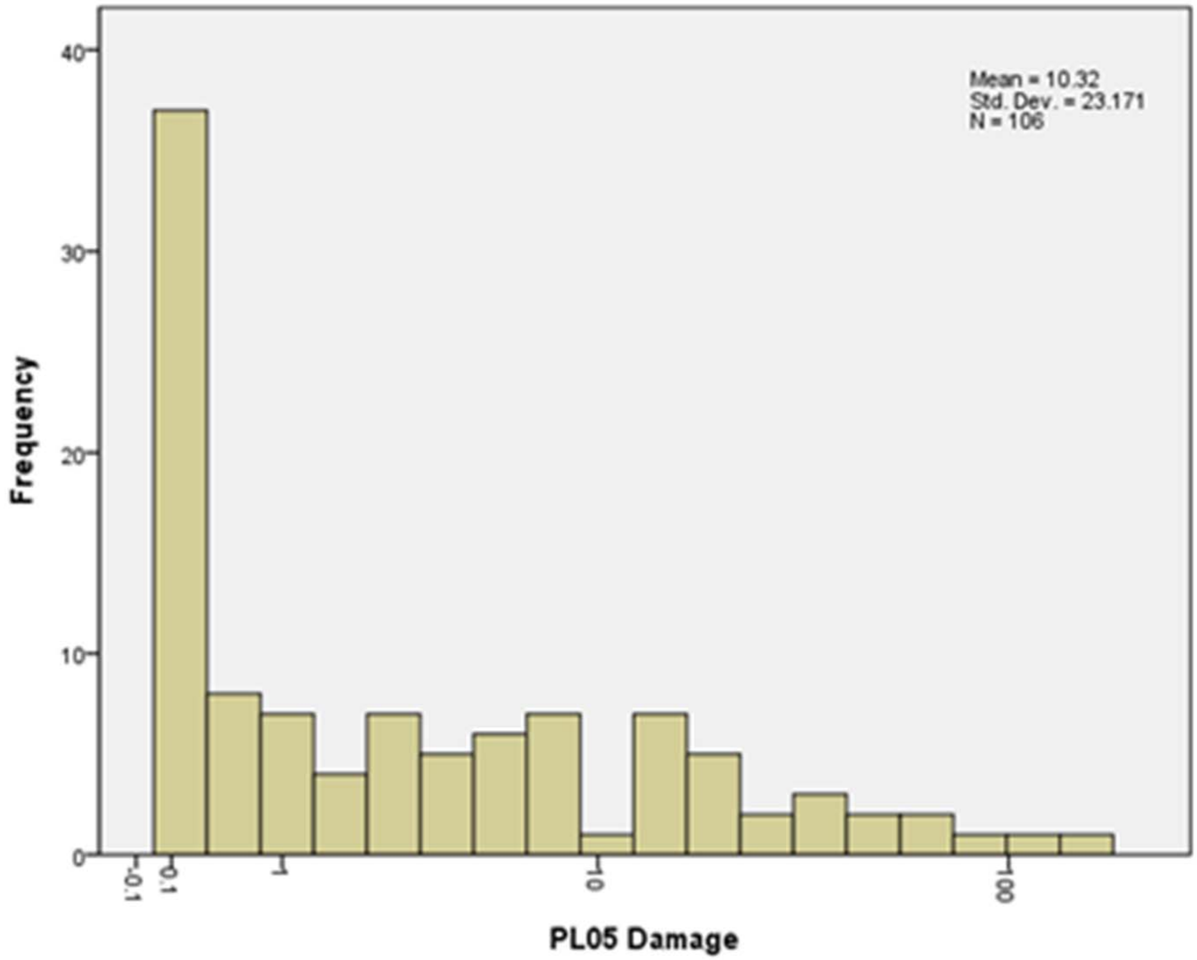
Descriptive Statistics

	N	Min	Max	Mean	Std. Dev.
PL05 Damage	106	0	161	10.3	23.2
CL05 Damage	106	0	144	10.1	21.8

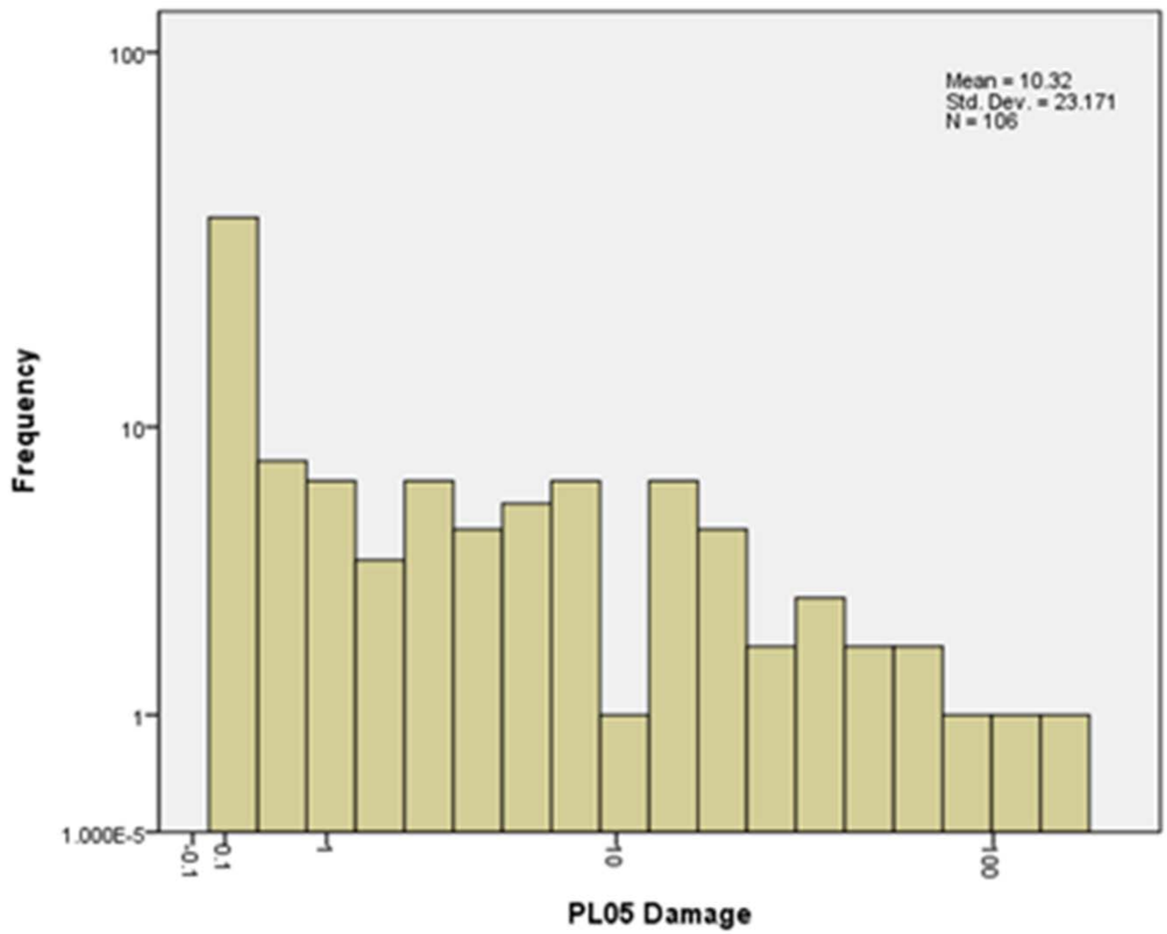
Histograms – Untransformed Data



Histogram - Log(Damages) Scale



Histogram - Log-Log Scale



Power Law Estimation

- To estimate ζ, use Hill's MLE:

$$\hat{\zeta} = 2 + n\left[\sum_{i=1}^{n} \ln\left(\frac{x_i}{x_{min}}\right)\right]^{-1}$$

- To calculate x_{min} follow suggestions of Clauset, Shalizi & Newman (2009):
  - Estimate ζ for each possible x_{min} and find:

$$KS = \max_{x \geq x_{min}} |G(x) - P(x)|$$

  - Choose x_{min} with smallest KS value.

Estimated Power Law Distributions

Variable	**Coefficient**	**Estimate**
PL	zeta	1.2185
	95% CI	0.9865, 1.4504
	xmin	12.9
CL	zeta	1.1408
	95% CI	0.9236, 1.3580
	xmin	11.7

Goodness of Fit Tests for Power Law Distribution (Clauset et al., 2009)

1. Generate many synthetic data sets from estimated PL distribution.
2. Re-estimate x_{min} and ζ for each synthetic data set.
3. Calculate a KS statistic for each synthetic data set by comparing to true distribution.
4. Count the proportion of times the original KS statistic exceeds the KS statistic from the synthetic data sets – proportion is p-value.

K-S Tests of Distributions

Variable	**Hypothesized Distribution**	**p-value**
PL	Normal	0.000
	Exponential	0.000
	Log-Normal	0.799
	Power Law	0.414
CL	Normal	0.000
	Exponential	0.000
	Log-Normal	0.760
	Power Law	0.156

Future Work

- Likelihood ratio tests to directly compare goodness of fit.
  - Nested hypothesis tests for tail.

Conclusion

- Hurricane damages may follow a power law or a lognormal distribution.
- If power law, variance of damages is estimated to be unbounded.