# Testing Predictions in Weighted Networks with the Dirichlet Covariate Model[†]

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**Abstract**

Recently, a wide variety of economic, social, and political settings are represented by models with *weighted networks* where individuals allocate resources across their different connections. The data associated to these settings is compositional (i.e., a point in a simplex). We introduce the Dirichlet Covariate Model (Campbell and Mosimann, 1987) for the empirical analysis of this compositional data. Thus, we contribute to the empirical analysis of networks; so far focused on *unweighted networks*. To illustrate the use of the method, we run a novel experimental design to test the equilibrium predictions of one of the untested models with weighted networks.

**Keywords**: Networks, Weighted Networks, Dirichlet distribution, Dirichlet Covariate Model, Multibattle Contest, Conflict, Experiments.

**JEL**: C91; D74; D91

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# 1 Introduction

No man is an island, entire of itself. Many economic, political and social interactions between individuals occur in a social structure, where individuals interact with each other. Network theory can formally represent individuals and social structures with nodes and links in a graph (see Jackson et al., 2008; Goyal, 2012; Bramoullé et al., 2016, for recent reviews). In a first wave of models with networks, individuals are allowed to select a single action affecting all their connections equally.[1] By doing so, we have learnt, for example, that particular social structures can help (or hinder) diffusion of opinions, technologies or a products (see Jackson and Yariv, 2011; Banerjee et al., 2017; Bloch et al., 2018, for some example). A more recent wave of models have expanded the framework allowing a) connections between individuals to have different importance (weighted networks) and b) individuals to select a distinctive action for each connection.[2] In these models, individuals usually allocate a finite resource (e.g., money, effort, attention) across the different connections (e.g., Goyal et al., 2008; Franke and Öztürk, 2015; Bourlès et al., 2017; Parise and Ozdaglar, 2019). Such models also have important and ubiquitous applications. For example, the allocation choices of resources across multiple group activities can drive the outcomes of *R&D* collaboration agreements, research in co-authorship networks, public goods in communities or even the outcome of international conflicts.

How do we analyze the data arising in these richer settings and test predictions from these recent theoretical models? In the absence of a method to deal with such data, a growing empirical literature has been restricted to analyze data and test models from the first wave, investigating how the topology of unweighted networks affects single choices (see Kosfeld, 2004; Choi et al., 2016; Jackson et al., 2017, for some reviews).[3] In this literature, the emerging univariate data can be analyzed with common econometric methods. However, in the settings associated to the second wave, the data is compositional, represented by the vector of the relative allocation by an individual across connections. Thus, we require a different method to analyze the compositional data emerging in settings from the second wave.

In this paper, we introduce the Dirichlet Covariate Model (Campbell and Mosimann, 1987) to an-

[1] There is also a literature on how social structures emerge (see Vannetelbosch and Mauleon (2016) for a review of endogenous network formation) but it is outside the scope of the current paper.

[2] There are some hybrid models – e.g., models where individuals choose a single action common to every link in a weighted network (DeMarzo et al., 2003; Ballester et al., 2006).

[3] Most of this empirical work is experimental and reviewed in Kosfeld (2004) and Choi et al. (2016). Despite common shortcomings with laboratory experiments (Falk and Heckman, 2009), laboratory experiments can induce exogenous variations to establish causal inferences, addressing concerns arising with naturally occurring data (e.g., endogeneity of the network structure). More details appear in section 4.3.2.

alyze such data. The Dirichlet distribution has been typically used in the analysis of compositional data in fields such as geology, machine learning, forensics and linguistics – prominent examples are Pawlowsky-Glahn and Buccianti (2011), Blei et al. (2003), Lange (1995), MacKay and Peto (1995), respectively. Three properties of the Dirichlet distribution and the Dirichlet Covariate Model are key.[4]

First, like a vector of allocations, the Dirichlet distribution lives in a constrained multivariate space: the simplex of dimension $\mathcal{D}-1$, where $\mathcal{D}$ also represents the dimension of the allocation vector to be analyzed.[5] Second, the Dirichlet distribution is characterized by $\mathcal{D}$ (concentration) parameters, one for each dimension. The expected value of a dimension is determined by the relative weight of the parameter associated to that dimension with respect to the sum of all of them. Thus, the marginal effects of the parameters acquire a transparent interpretation – a change in the relative weights of each allocation. In addition, the variance is determined exclusively by the same concentration parameters. Finally, these concentration parameters can be expressed as a linear function of a set of covariates, which can be estimated via maximum likelihood. Once this econometric model is estimated, Wald tests can be used to test hypotheses – e.g., the risk neutral equilibrium predictions across connections in weighted networks.[6]

To illustrate the use of the Dirichlet Covariate Model, we select one of the untested models from the second wave. In particular, we design an experiment implementing a variation of a model of bilateral conflicts in weighted networks. This model builds on Franke and Öztürk (2015) and has been generalized by Cortés-Corrales and Gorny (2018).[7] In the model, each agent decides simultaneously how to allocate their budget across conflicts with other agents they are connected. Each conflict has an associated value and the winner is selected stochastically *à la* Tullock (1980). The objective of the game consists on maximizing the expected payoffs.

In this model, the equilibrium is unique, interior and in pure strategies for a wide range of parameters. By implementing parameters in this range, our experiment eliminates potential confounds

[4] Another interesting property of the Dirichlet distribution (i.e., the prior and posterior are conjugate distributions) is useful to estimate priors in Bayesian methods (e.g., Conley et al., 2008; Jensen and Maheu, 2010).

[5] The proposed method can be also applied to alternative strategy sets as long as their domain can be transformed into the simplex. For example, a multidimensional strategy set representing efforts across connections does not need to be restricted to live in the simplex (i.e., efforts do not need to sum up to 1). However, these efforts will need to be transformed into the relative efforts, which sum up to 1, and, then, our method could be applied.

[6] Testing predictions in weighted network is indeed scarce. According to Choi et al. (2016), "... there is no paper we are aware of in the network experiments literature within and outside of economics which has investigated weighted networks. The creation of weighted networks in the lab presents its own challenge..." (p. 468).

[7] Related models of bilateral conflicts are Huremovic (2015), Hiller (2016), König et al. (2017) and Dziubinski et al. (2017). Konrad (2009) surveys related multi-battle models without networks and their applications, such as R&D races, political elections, marketing strategies, sports competitions or war between countries, amongst others. For the corresponding survey of the experimental literature, see Dechenaux et al. (2015).

that could emerge – e.g., participants playing different equilibria or the natural inability of participants to mix strategies optimally (Walker and Wooders, 2001; Wooders, 2010). Interestingly, this unique and interior equilibrium in pure strategies of the implemented model is proved to be impossible to characterize in general, using algebraic operations and roots of natural degrees. When four players or more are connected in a path, finding the equilibrium can be reduced to solving a general (and irreducible) polynomial of degree 8 or higher. According to the Abel-Ruffini Theorem, this class of polynomials cannot be solved using radicals. Thus, not only participants in an experiment but *any* human, including game theorists, need to tolerate some error level in approximating the equilibrium with their choices.[8]

Using the Dirichlet Covariate Model, we can establish that, in the experiment, there are statistically significant deviations from equilibrium in weighted networks. These results extend the existing evidence of deviations in unweighted networks (Choi et al., 2016), which could not be possible without the Dirichlet Covariate Model.

In addition, to understand how aspects of the topology of the network affect deviations, our experimental design varies systematically three elements of the network: own degree, the distribution of the degree of the co-participants and weights of the connections.[9] These particular elements are selected because they trace back to cognitive limitations.[10] In our experiment, by increasing a participant's degree, their strategy space becomes larger and, consequently, finding optimal solutions is more challenging. By increasing the diversity in the degree of the co-participants, a participant faces different cognitive types (i.e. participants with different strategy space size), which requires a higher strategic sophistication to find the optimal. By changing the weights of the connections from symmetric (all connections are valued the same) to heterogeneous (connections are valued differently), the optimal solution becomes computationally more demanding. The results show that every aspect of the topology affect the deviations but only own-degree is significant. An increase from degree 2 to degree 3 translates into a relative increase of the deviation from equilibrium (between 35% and 43.75%).

The rest of the paper is structured as follows. Section 2 introduces the Dirichlet Covariate Model. Sections 3 and 4 describe the theoretical model of bilateral conflict and the experimental design,

[8] In fact, equilibrium allocations in this paper will refer to approximations found through the Newton-Raphson algorithm whose tolerance is $10^{-12}$ in most popular statistical packages. The deviations exhibited by the actual subjects are substantial enough for this level (or lower levels) of tolerance not to be binding.

[9] Previous papers include a variety of networks to show how deviations are robust across networks topologies but the variation is not designed to study the causal effect of aspects of the topology on deviations. However, correlations can still be established – e.g., Gallo and Yan (2015) establishes correlations between degree and observed actions.

[10] Few models with networks have started to investigate the effects of the cognitive limitations of the agents (e.g. Choi, 2012; Choi et al., 2012; Dessi et al., 2016).

respectively, which has been selected to illustrate the use of the Dirichlet distribution. Section 5 includes the experimental results and Section 6 concludes.

# 2 The Dirichlet Covariate Model to Test Predictions in Weighted Networks

There are alternative methods to the Dirichlet Covariate Model in order to analyze compositional data. A prominent alternative is the additive log-ratio transformation (Aitchison, 1982), such that the constrained properties of the simplex space are removed by projecting the data into the multivariate real space. Consequently, conventional multivariate techniques are again applicable. However, the marginal effects in terms of the original ratios are not possible to recover and, what is more, the interpretation of the estimated marginal effects with the log-ratio transoformation becomes difficult in general.[11] This said, an exception to this issue has allowed this method to enjoy certain prominence within economics – more concretely, in the parametric analysis of demand (see, for example, Deaton and Muellbauer, 1980; Banks et al., 1997; Lewbel and Pendakur, 2009). In this literature, the researcher derives specific but flexible demand functions, satisfying axioms of decision theory; the marginal effect of the log transformation of prices, $log(p)$, on quantity, $log(q)$, can be interpreted as the price elasticity of demand.

However, the estimation and interpretation of marginal effects with this method in other settings become difficult. For example, in strategic settings, beliefs as well as preferences are included in a rational decision. Thus, the log-ratio transformation method does not enjoy the same possibility of interpreting the marginal effect in strategic settings as in the parametric demand analysis. In addition, recent research (Hijazi and Jernigan, 2009) have shown that, for compositional data, the Dirichlet distribution produces estimates closer to the real ones than the log-ratio transformation method.

For these reasons, we propose to use, instead, the Dirichlet Covariate Model (Campbell and Mosimann, 1987). Thus, we assume the Dirichlet distribution as the generating process of the compositional data. The Dirichlet distribution is endowed with interesting properties, making this

[11] To illustrate this point, consider the relation between two variables $Y$ and $X$, such that $Y = g(X)$ and $Y$ is a compositional variable, $Y = \dfrac{a}{a+b}$ with $(a+b) = 1$. The log-ratio transformation of $Y$ implies $\ln(Y) = \ln(a) - \ln(a+b) = \ln(a)$. Thus, by projecting $Y$ to the unconstrained multivariate space, we estimate $\ln(a) = g(X)$. The estimates function $\hat{g}(X)$ reflects the marginal effect of $X$ on $\ln(a)$, but not on $Y$. Hence, this transformation loses information about the marginal effect of $X$ on $Y$. It is always possible, however, to estimate structural "deep" parameters. Choi et al. (2007) is a recent example in which risk and disappointment aversion parameters are estimated.

distribution a natural candidate. Furthermore, the parameters of the distribution can be estimated via maximum likelihood as a linear function of some observables. Once the model is estimated, we can test *any* specific point prediction (e.g., equilibrium predictions) with Wald tests.

Suppose a multivariate random variable $\boldsymbol{y} = (y_1, \ldots, y_{\mathcal{D}})$ of dimension $\mathcal{D} \geq 2$, whose component $y_k$ $\forall k$ is a real valued random variable. We denote $\boldsymbol{y} \sim Dir(\boldsymbol{\alpha})$ when $\boldsymbol{y}$ follows a Dirichlet distribution of parameters $\boldsymbol{\alpha}$. The probability density function of a Dirichlet distribution of parameters $\boldsymbol{\alpha}$ is given by

$$f(y_1, \ldots, y_{\mathcal{D}}; \alpha_1, \ldots, \alpha_{\mathcal{D}}) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{k=1}^{\mathcal{D}} y_k^{\alpha_k - 1}, \tag{1}$$

where the vector $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_{\mathcal{D}})$ gives the *concentration parameters* shaping the distribution. The normalizing constant $B(\boldsymbol{\alpha}) = \frac{\prod_{k=1}^{d} \Gamma(\alpha_k)}{\Gamma(\sum_{k=1}^{d} \alpha_k)}$ is the multivariate Beta distribution (the conjugate prior of the binomial), which can be expressed using the Gamma distribution, $\Gamma(.)$. This constant ensures that function (1) results into a probability density function with total probability of one.

The Dirichlet distribution possesses the following properties. First, the support of the Dirichlet distribution is the $(\mathcal{D} - 1)$-dimensional simplex,

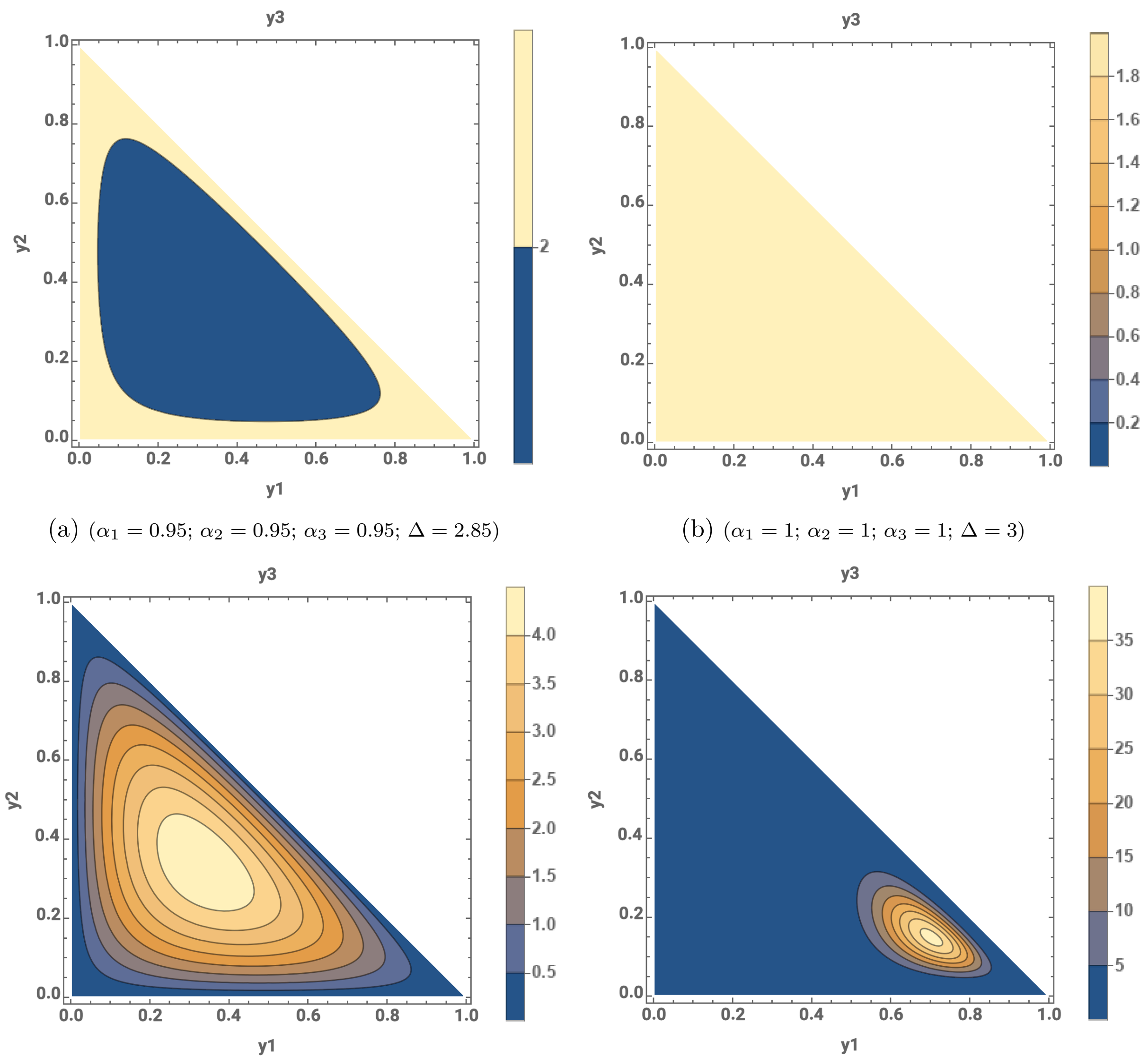
$$\mathbb{S}^{\mathcal{D}} = \{(y_1, \ldots, y_{\mathcal{D}}) \mid \sum_k y_k = 1, y_k > 0 \text{ for } k = 1, \ldots, \mathcal{D}\}. \tag{2}$$

Second, when $\boldsymbol{y} \sim Dir(\boldsymbol{\alpha})$, the expected value of the element $y_k$ is given by the relative weight of the concentration parameters associated to that element with respect to the sum of all the concentration parameters, $E[y_k] = \frac{\alpha_k}{\Delta}$, where $\Delta = \sum_{k=1}^{\mathcal{D}} \alpha_k$. Thus, the interpretation of the marginal effect of any concentration parameter has a direct interpretation – a change in the relative weights of each element. Note that, because we are working in the simplex, an increase of one element necessarily implies a decrease in the same proportion of at least one other element.

Finally, the variance of the element $y_k$ is also defined *exclusively* in terms of the concentration parameters, $Var(y_k) = \frac{\alpha_k(\Delta - \alpha_k)}{\Delta^2(\Delta + 1)}$. Thus, expected value and variance can be determined jointly via the concentration parameters. One important aspect to notice is that larger concentration parameters express a higher concentration around the expected value.

Figure 1 illustrates these properties by showing how the shape of the Dirichlet distribution changes as their concentration parameters change. All these examples are in the 2-dimensional simplex. In panels (a), (b), and (c), $E[y_k] = 1/3$ $\forall k$. Thus, the distributions are centered at the vector of expected values $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. The only difference between these three panels is the levels of the concentration parameters – $(0.95, 0.95, 0.95)$, $(1, 1, 1)$ and $(2, 2, 2)$ for panels (a), (b), and (c), respectively. This change in level, which does not affect the expected value, decreases the variance of each element. Thus, in panel (a), as $\Delta < \mathcal{D}$, points far away from the vector of expected values are assigned a higher probability than points closer to the mean. In panel (b), as $\Delta = \mathcal{D}$, every point in the simplex is assigned the same probability, which leads to the multivariate uniform distribution. In panel (c), as $\Delta > \mathcal{D}$, points closer to the vector of expected values are assigned a higher probability than points away from the mean. Unlike the previous panels, panel (d) shows the distribution centered at $(\frac{2}{3}, \frac{1}{6}, \frac{1}{6})$ with a very small variance.

Figure 1: Examples of Dirichlet Distributions.




(a) ($\alpha_1 = 0.95$; $\alpha_2 = 0.95$; $\alpha_3 = 0.95$; $\Delta = 2.85$)

(b) ($\alpha_1 = 1$; $\alpha_2 = 1$; $\alpha_3 = 1$; $\Delta = 3$)

(c) ($\alpha_1 = 2$; $\alpha_2 = 2$; $\alpha_3 = 2$; $\Delta = 6$)

(d) ($\alpha_1 = 20$; $\alpha_2 = 5$; $\alpha_3 = 5$; $\Delta = 30$)

The objective is to analyze compositional datasets. Formally, a compositional dataset $\boldsymbol{\mathcal{Y}}$ with $T$

observations is defined as follows $\boldsymbol{\mathcal{Y}} = (\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_T)$, where $\boldsymbol{Y_t} = (Y_{t1}, \ldots, Y_{t\mathcal{D}}) \in \mathbb{S}^{\mathcal{D}}$. The unit of observation could be a period, an individual, or both. In this paper, the generating process behind this dataset is assumed to be a Dirichlet distribution, which includes the aforementioned properties.

The concentration parameters associated to a Dirichlet generating process can be conditioned on a set of observed covariates, $\boldsymbol{x} = (x_1, \ldots, x_c) \in \mathbb{R}^c$ such that $\boldsymbol{\alpha}(\boldsymbol{x}) = (\alpha_1(\boldsymbol{x}), \ldots, \alpha_{\mathcal{D}}(\boldsymbol{x}))$. Campbell and Mosimann (1987) propose a linear function where $\alpha_k(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{\beta}_k$ with $\boldsymbol{\beta}_k$ being a $(c \times 1)$ vector of estimated coefficients. Given a dataset of covariates $\boldsymbol{\mathcal{X}} = (\boldsymbol{X}_1, \ldots, \boldsymbol{X}_T)$, we need to estimate the vector of parameters $\hat{\boldsymbol{\beta}} = (\hat{\boldsymbol{\beta}}_1, \ldots, \hat{\boldsymbol{\beta}}_{\mathcal{D}})$ subject to the constraint that every $\hat{\alpha}_k(\boldsymbol{X}_t) > 0$.[12] These constrains ensure that $\mathcal{D}$ is the dimension of the simplex of the estimated model. Assuming that the observations, $\{\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_T\}$, are i.i.d. given $\boldsymbol{\beta}$, the likelihood function for the observed sample can be written as

$$\mathcal{L}(\beta|\boldsymbol{X}, \boldsymbol{\mathcal{Y}}) = \prod_{t=1}^{T} \left[ \Gamma(\sum_{k=1}^{\mathcal{D}} \alpha_k(\boldsymbol{X}_t)) \prod_{k=1}^{\mathcal{D}} \frac{Y_{tk}^{\alpha_k(\boldsymbol{X}_t)-1}}{\Gamma(\alpha_k(\boldsymbol{X}_t))} \right].$$

The relevant parameters can be estimated via maximum likelihood estimation.[13] Once this Dirichlet Covariate Model is estimated, any theoretical point prediction, $\boldsymbol{a}^*$, can be formally tested by using Wald tests for composite hypotheses over $\hat{\boldsymbol{\alpha}} = (\hat{\alpha}_1, \ldots, \hat{\alpha}_{\mathcal{D}})$. The lack of independence between the estimated alphas prevent us from testing them separately. Thus, point predictions need to be translated into joint hypotheses about the estimated alphas. In the simplex, there is clearly a one-to-one correspondence between point predictions and composite hypotheses – e.g., the prediction $\boldsymbol{a}^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ and the joint hypothesis $\hat{\alpha}_1 = \hat{\alpha}_2 = \hat{\alpha}_3$; the prediction $\boldsymbol{a}^* = \left(\frac{2}{3}, \frac{1}{3}\right)$ and the joint hypothesis $\hat{\alpha}_1 = 2\hat{\alpha}_2$.

Network models in the second wave describe settings producing compositional datasets. Thus, the Dirichlet Covariate Model can be used to analyze these settings and test equilibrium predictions. The only concern is that the support for the Dirichlet distribution is a proper subset of the strategy spaces in these models (and the corresponding empirical observations). While the latter includes the boundary of the simplex (i.e., some elements of the allocation vector might be zero), the former requires a positive mass in every dimension – see expression (2). This problem also arises with the log-ratio transformation method. In fact, we can apply any of the three solutions regularly applied in that method. The first solution is to drop observations in the boundary. The second solution is

[12] Thus, the total number of parameters to be estimated is $c \times \mathcal{D}$.

[13] The estimated parameters in the maximization problem of the maximum likelihood estimation method are equivalent to those of the minimization problem of the negative log transformation of the above likelihood expression. As it is common practice, for computational purposes, we proceed with the minimization problem. Furthermore, following Campbell and Mosimann (1987), we also transform $\alpha_k(\boldsymbol{X}_t)$ with the log-link function $g(.)$ in order to meet the constraint that $\hat{\alpha}_k(\boldsymbol{X}_t) > 0$.

to perturbate boundary observations with an $\epsilon$ change such that the observation becomes interior. The final solution is to transform the observation to a different space, by "compressing" the data in each dimension symmetrically around $\frac{1}{D}$, producing biased but consistent estimators (further details in Smithson and Verkuilen, 2006).

To illustrate the use of the Dirichlet Covariate model, we select a variation of one of the theoretical models from the second wave and design a laboratory experiment implementing it. Although the Dirichlet Covariate Model could also be used with naturally occurring data,[14] we opted for such experiment because, as mentioned in footnote 6, experimental work in exogenous weighted network appears to be missing. In particular, to the best of our knowledge, none of the models in the second wave has been tested.

## 3 Selected Model

A social structure comprises a finite set of agents $N = \{1, 2 \ldots, n\}$ and a set of connections $B$ whose elements are unordered pairs of agents. We can represent this structure with a graph $\mathcal{G} = (N, B)$. In this graph, agent $i$ is represented by a node and a connection between agents $i$ and $j$ is represented by a link if and only if $(ij) \in B$. In our model of conflict, if $i$ and $j$ are linked, we say that $i$ has a conflict with their rival $j$. Every rival $j$ of $i$ belongs to the set of rivals, $j \in N_i$. The own-degree (or degree) of agent $i$, $d_i$, is given by the cardinality of the set of rivals, $d_i = |N_i|$. At this point, we notice three constraints. First, all conflicts between a pair of agents are summarized into a *single* conflict (or lack of it), avoiding parallel links and multigraphs. Simply put, agent $i$ cannot have more than one conflict with agent $j$. Second, in the model, agents are excluded from being their own rivals. Hence, $\mathcal{G}$ needs to be irreflexive (i.e., $(ii) \notin B$). Finally, agents cannot ignore a conflict ex ante.[15] These mutual conflicts induce $\mathcal{G}$ to be undirected: if $(ij) \in B$, then $(ji) \in B$ $\forall i \neq j$.

In this social structure, every agent $i$ is endowed with a budget $w_i > 0$ of a uni-dimensional resource (e.g., effort, money). Each agent simultaneously decides how to allocate their budget across their conflicts, determined by the graph $\mathcal{G}$. Any fraction of the budget not allocated is lost. Thus, an agent's pure strategy is given by an allocation vector $\mathbf{s}_i$ of dimension $d_i$, where the typical element $s_{ij}$ represents the allocation of agent $i$ to their conflict with their rival $j \in N_i$, such that

[14] We are unaware of such datasets. By proposing this method, we hope to encourage others to collect such datasets and use the Dirichlet Covariate Method for their analysis.

[15] However, this does not prevent agents from "ignoring" a conflict ex-post by allocating no resources to a particular conflict, while their rival allocates a positive amount. While this is possible in the general model, the focus of this paper is on parameters inducing interior optimal allocations.

$\sum_{j\in N_i} s_{ij} = w_i$. For the rest of the paper, this allocation vector is normalized with respect to the budget (i.e., $a_{ij} = \frac{s_{ij}}{w_i}$). Thus, we obtain a sum-unit vector $\mathbf{a}_i$, such that $\sum_{j\in N_i} a_{ij} = 1$ and $a_{ij} \geq 0$. This normalization does not affect the optimal strategy but will allow the use of the Dirichlet distribution in the empirical analysis.

The value associated to winning the conflict in which agents $i$ and $j$ are involved is $v_{ij}$ and the value associated to losing the conflict is normalized to 0. We assume that every pair of rivals, $i$ and $j$, agree on the value of their conflict, i.e., $v_{ij} = v_{ji}$, and that these values are common knowledge in the social structure. Furthermore, let $v_{ih} = max\{v_{ij}|j \in N_i\}$ be agent $i$'s most valued conflict and $v_{il} = min\{v_{ij}|j \in N_i\}$ be agent $i$'s least valued conflict.

The winner of each conflict is determined by a lottery *contest success function* (Tullock, 1980) such that the relative probability of agent $i$ winning the conflict against agent $j$ is given by:

$$p_{ij} = \begin{cases} \frac{a_{ij}}{a_{ij}+a_{ji}} & \text{if} \quad (a_{ij} + a_{ji}) \neq 0 \\ \frac{1}{2} & \text{if} \quad a_{ij} = a_{ji} = 0. \end{cases}$$

Then, the objective of every agent $i$ is to maximize the sum of the values of the conflicts won. Thus, each agent $i$ is facing the following $d_i$ dimensional constrained maximization problem.

$$\max_{\mathbf{a}_i} \quad \pi_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}, \mathcal{G}) = \sum_{j\in N_i} v_{ij} p_{ij} \quad \text{subject to} \quad \sum_{j\in N_i} a_{ij} = 1$$

Now, we claim.

**Proposition 1.** *In this setting, there exists a unique interior equilibrium in pure strategies if*

$$\frac{v_{il}}{v_{ih}} > \frac{1}{4} \quad , \quad \forall i \in N$$

The proof appears in Appendix A.1 (see Franke and Öztürk, 2015; Cortés-Corrales and Gorny, 2018, for results in similar games with convex cost functions).

In this paper, we focus on sets of parameters where the condition of Proposition 1 applies and the equilibrium is unique, interior and in pure strategies. The next step is to address issues about the characterization of this equilibrium. For that, we define first a path in a network $\mathcal{G}$.

**Definition.** *A path in a network $\mathcal{G}$ between agents $u$ and $v$ is a finite sequence of connections $\{i_1i_2, \ldots, i_{k-1}i_k\}$ and a set of distinct agents $\{i_1, \ldots, i_k\}$ such that every $i_m i_{m+1} \in B$ for each $m \in \{1, \ldots, k-1\}$, with $i_1 = u$ and $i_k = v$.*

In this game, the optimal allocation of agent $i$ to the conflict with agent $j$ depends on the allocation of agent $j$, which, at the same time, depends on the allocations of the other agents connected to $j$, which, in turn, depend on the allocation of subsequent agents connected to agents connected to $j$ and so on. This interdependence of the best-responses between players in the network is determined by the different paths. Given the contest success function, this interdependence is non-linear and it is determined by the length of the paths in the network. The relationship between characterization of equilibrium and length of the path is expressed in the following Proposition.

**Proposition 2.** *A* generic *explicit characterization of the equilibrium, using algebraic operations and roots of natural degrees, does not exist if there is a path of length equal or higher than* 3 *in the network.*

The proof appears in Appendix A.2. In the proof, we show that when there exists a path of length three, in general, finding the equilibrium can be reduced to solving a general (and irreducible) polynomial of degree 8. Longer paths lead to general polynomials of higher degree. However, according to the Abel-Ruffini Theorem, this class of polynomials cannot be solved using radicals. Therefore, the equilibrium characterization using algebraic operations and roots of natural degrees is not possible in general. Notice that other elements of the network topology, including the different measures of centrality, are not necessary to reach this result.

We should make clear, however, that Proposition 2 does not preclude from finding the characterization of a solution in particular cases. In fact, certain symmetries in the topology, and the distribution of conflict values allow the polynomial to be further simplified (e.g., into quadratic polynomials), providing solutions for particular cases. For example, in $q$-regular networks, where every agent has the same $q \in \{2, ..., n-1\}$ numbers of conflicts (i.e., $d_i = q \;\; \forall i \in N$), if $\sum_{j \in N_i} v_{ij} = \bar{V}$ for every $i \in N$, the equilibrium allocation by agent $i$ to conflict $j$ is given by $a^*_{ij} = \frac{v_{ij}}{\bar{V}}$ (see proof in Appendix A.2).

Whenever such simplifications are not possible, one shall resort to computational methods. In particular, for the rest of the paper, we use the Newton-Raphson algorithm to find the roots of the polynomials resulting from the set of first order conditions of every player in every conflict.

This method uses a sequences of tangent lines to approximate the roots of the polynomial. A pre-specified level of tolerance determines when the algorithm is close enough and should stop. Most popular statistical packages will allow a maximum tolerance of $10^{-12}$ for the Newton-Raphson algorithm.

Consequently, the fact that computers as well as *any* human (and not only experimental subjects) need to accept a level of error when choosing a strategy appears to be an inescapable truth in our setting.

# 4 Experiment

## 4.1 Experimental Games and Treatments

Our experimental games use the same structure as the games described in the previous section. When implementing these experimental games, we include the following two restrictions common to all treatments. The first common restriction is that every participant is endowed with $w_i = w = 100$ tokens, which participants need to allocate to different conflicts. By normalizing this endowment to 100, the relative share of the endowment allocated by participant $i$ to conflict $j$, $a_{ij}$, is easy to calculate. The second common restriction is that the number of participants is always $n = 4$. This is the smallest number of participants ensuring that there exists a path of length 3 in the networks of our experimental games and, consequently, Proposition 2 is satisfied.

In addition to these common restrictions, our experiment uses various restrictions on the network structure and the values of the conflicts to create the different experimental treatments. We present first the treatments arising from the variation in the network structure. In our experiment, we consider the four connected networks shown in Figure 2: *Complete*, *Ring*, *Diagonal* and *Line*. Each network contains four nodes, representing the participants, and the corresponding links, representing the implemented conflicts between participants. As observed in Figure 2, the degree of a participant can be either 1, 2 or 3, depending on the network and position. In this experiment, we focus on participants with degree 2 (*Degree 2 treatment*) and degree 3 (*Degree 3 treatment*) because participants with degree 1 have no strategic decision to make.[16] By increasing the degree of the participants, our experimental treatments can compare, other things equal, how the size of the strategy space affects strategic choices in networks.

[16]In our experiment, they are simply required to submit their whole endowment to the only conflict available to them. Further details in section 4.2.

So far, degree is the main observable characteristic of participants in the network. If this characteristic is taken as an induced type, the second set of treatments refer to the changes in the distribution of the types within a network. In the top row of Figure 2, the only two $q$-regular networks with four participants are shown (the Complete and the Ring). In these networks, all participants are induced the same type, either degree 2 or degree 3 (*Identical Treatment*). The bottom row includes the Diagonal and the Line, which are created by removing a link from the Complete and the Ring, respectively.[17] In these two network, there is diversity of types (*Diverse Treatment*). Participants in the Diagonal have either degree 2 or 3 and participants in the Line have either degree 1 or 2.

Equilibrium play and best-responses become computationally more difficult in the presence of diversity of types than in its absence. As shown in Proposition 2, best-responses depend on the paths in the network, connecting a player with any other players.[18] Thus, all connected types affect a player's best response and, consequently, the resulting equilibrium. In Identical treatments, all types are the same and it turns out that simple heuristics (e.g., splitting your endowment equally across conflicts, $a_{ij} = \frac{w}{d_i}$, or proportional allocations with respect to the relative values, $a_{ij} = w\frac{a_{ij}}{\sum_{j \in N_i} a_{ij}}$) are observationally equivalent to equilibrium play (see Table 1). This is not the case in Diversity treatments where equilibrium produces predictions, which do not seem to be equivalent to any obvious heuristic. Since the construction of the best-responses in our model appears to be independent of other elements of the network (for example, measures of centrality), we abstract from such elements in the experiment for the remainder of the paper.[19]

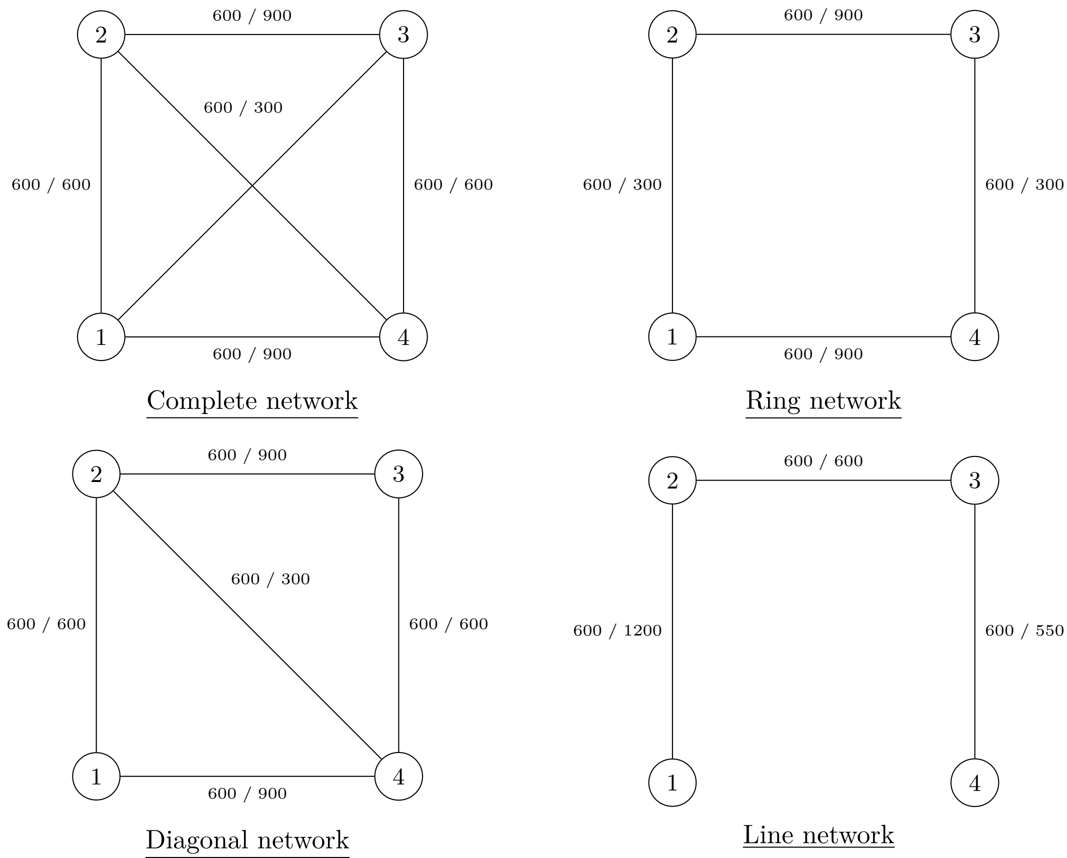
Finally, the last variation of our experimental design refers to changes in the values of the conflicts. In particular, we have two treatments: *Homogeneous treatment* and *Heterogeneous treatment*. Figure 2 shows the number of points associated to each conflict in the two treatments. In each link, the number to the left and to the right refers to the value in the Homogeneous and the Heterogeneous

[17] In addition, there are two other networks in the set of connected non-isomorphic networks with four nodes: the *Star* and the *Kite*. In the Star network, only one out of four participants can exhibit strategic considerations while the other participants should trivially allocate their budget to their only conflict. The Kite network is the only non-isomorphic network of four participants with three types, making comparison with the other networks relatively more difficult.

[18] Against Proposition 2, one could assume that participants lack complete information in the experiment and only consider their degree or paths of length 1 (see Charness et al., 2014, for the use of incomplete information in networks). In this case, the definitions of the Identical and the Diverse treatments would have slightly different implications. For example, participants 1 and 3 in the Diagonal would be included in the Identical treatment rather than the Diverse treatment because both their immediate rivals are of the same type. We checked how robust the results of our preferred (treatment) definitions (presented in section 5) are in the face of these alternative definitions (presented in the Online Appendix). The results of our preferred definitions are similar but more conservative (i.e., fewer treatments are significant) than alternative definitions.

[19] In addition to best-responding to equilibrium actions, the Online Appendix also includes the possibility that participants best-respond to the empirical distribution and just to the past observed play. Furthermore, we relax the assumption of best-responses and analyze the use of simple decision rules related to the structure of the network. The results presented in this paper are robust to these alternative benchmarks.

Figure 2: Selected Networks (*Homogeneous*/*Heterogeneous* values).




treatment, respectively. In the Homogeneous treatment, every conflict in every network is worth the same number of points, $v_{ij} = 600\ \forall i, j$. Thus, every rival and the associated conflict is only distinguishable in terms of the changes in the network structure described so far.

For any given network, the Heterogeneous treatment introduces differences in the number of points across conflicts. A different number is drawn for each conflict of a participant from the following set $v_{ij} = \{300, 550, 600, 900, 1200\}$ such that, in every network, the highest number of points for a participant is not larger than four times their lowest number. By doing so, Proposition 1 is satisfied and every weighted network contains an interior equilibrium, different from the equilibrium in the Homogeneous treatment (see Section 4.3). Thus, in the Heterogeneous treatment, participants might not only face a larger strategy space, and diversity of types but also a different weight for each conflict, which would make calculations even more complicated.

## 4.2 Implementation of the Experimental Design

In an experimental session, each participant is allocated at random to one node and network. This allocation remains the same throughout the experiment, imposing a partner matching. At the beginning of the experiment, instructions are read aloud (a copy can be found in Appendix B.1). In the instructions, participants are told that there are two stages and that they only receive specific information about the second stage (Heterogeneous treatment) once the first stage (Homogeneous treatment) is finished. In each stage, participants play 20 rounds of the experimental game described above and feedback is provided between rounds.[20] At the end of the experiment one round per stage is selected at random for payment.

Participants face interfaces adjusted to their degree, created with z-Tree Fischbacher (2007) – screenshots of the corresponding interfaces appear in Appendix B.2. In particular, participants of degree 3 face a simplex of dimension 2 (represented by an equilateral triangle), where every point in the simplex represent a feasible allocation exhausting their budget; and each vertex represents a full allocation to one conflict.[21] Similarly, participants of degree 2 face a simplex of dimension 1 (represented by a slider). Finally, participants of degree 1 have no strategic role and are simply required to type 100 (i.e., their whole budget). All these interfaces force participants to exhaust their budget in each round and, therefore, strictly dominated strategies (not exhausting the budget) are ruled out as explanations for deviations from equilibrium predictions.

At the beginning of each round, a random point appears in the simplex of dimension 1 and 2. To choose an allocation, participants click on the point, drag it towards a new allocation and unclick it when they reach the allocation. They can relocate the point as many times as they wish. In addition to the graphical representation of the simplex, participants can see the number of tokens allocated to each conflict, which is updated every time participants select a new point in the simplex. To control for any possible order effects, the position of the vertices of the triangle and the slider are independently randomized in each round across participants.

The experiment was conducted at the Leicester EXperimental ECONomics (LExEcon) laboratory at the University of Leicester. Once participants arrived to the laboratory, they were seated at an individual computer workstation. A total of 236 participants drawn from the common participant pool participated in one of sixteen sessions. In the experiment, the exchange rate was £1 for every

[20] The equilibrium of this finitely repeated game is, effectively, playing, in each round, the stage equilibrium presented in Table 1.

[21] The granularity of this allocation is .1, which translates into a granularity of .001 when referring to the relative shares.

100 points. On average each session lasted about 80 minutes with an average payment of £16.1 (including £2 show-up fee).

## 4.3 Predictions

### 4.3.1 Equilibrium Predictions

The impossibility of expressing the equilibrium in general requires a level of tolerance to be assumed. For presentation purposes, the equilibrium predictions in terms of relative shares are rounded to .001. To compare predictions for individual participants within and across networks, isomorphic decision problems within a network are collapsed together by redefining relative allocation shares, $a_{i1}$, $a_{i2}$, and $a_{i3}$ for $i = \{1, 2, 3, 4\}$, such that a) $a_{i1}$ and $a_{i2}$ appear in every network, and $a_{i3}$ in the Complete and Diagonal network, and b) $a_{i1}$ corresponds to the highest valued conflict, $v_{ih}$, in the Heterogeneous treatment, and $a_{i2}$ (or $a_{i3}$ if it exists) corresponds to the lowest valued conflict, $v_{il}$.[22]

The corresponding predictions appear in Table 1. In Table 1, the left and the right column show the predictions in the Homogeneous and the Heterogeneous treatments, respectively; while each row represents a network. The row corresponding to the Diagonal network has two entries to distinguish participants with degree 2 ($i = \{1, 3\}$) and degree 3 ($i = \{2, 4\}$). Similarly, the row corresponding to the Line network also has two entries in the Heterogeneous column for participants $i = \{2\}$ and $i = \{3\}$ because of the different predictions in the Heterogeneous treatment.

### 4.3.2 Deviations from Equilibrium in Weighted Networks

Our experiment can establish to what extent lack of support for equilibrium, which has been reported in unweighted networks, also extends to weighted networks. In our experiment, the null hypotheses of equilibrium predictions are summarized in Table 1. The alternative hypothesis is summarized as follows.

[22] More concretely, we redefine the allocations shares as follows. In every network, $a_{i1}$ is the share that participant 1, 2, 3 and 4 allocates to the conflict with participant 4, 3, 2 and 1, respectively. $a_{i2}$ is the share that participant 1, 2, 3 and 4 allocates to the conflict with participant 2, 1, 4 and 3, respectively. And, $a_{i3}$ is the share that participant 1, 2, 3 and 4 allocates to the conflict with participant 3, 4, 1 and 2, respectively. In a slight abuse of notation for the Line network in the Heterogeneous treatment, let $a_{i1}$ is the share that participant 2 and 3 allocates to the conflict with participants 1 and 2, respectively. And $a_{i2}$ is the share that participant 2 and 3 allocates to the conflict with participants 3 and 4, respectively.

Table 1: Nash Equilibrium Predicted Relative Share

| | | Homogeneous | Heterogeneous |
|---|---|---|---|
| Ring | | (0.50,0.50) | (0.75,0.25) |
| Complete | | (0.333,0.333,0.333) | (0.500,0.333,0.167) |
| Line | $i=\{2\}$ | (0.535,0.465) | (0.672,0.328) |
| | $i=\{3\}$ | | (0.544,0.456) |
| Diagonal | $i=\{1,3\}$ | (0.50,0.50) | (0.60,0.40) |
| | $i=\{2,4\}$ | (0.328,0.328,0.343) | (0.499,0.332,0.166) |

In a cell, the vector ($a^*_{i1}$, $a^*_{i2}$) and ($a^*_{i1}$, $a^*_{i2}$, $a^*_{i3}$) corresponds to the relative shares of degree 2 and 3 participants, respectively.

**Hypothesis 1** (Deviations from Equilibrium)**.** *There are deviations from equilibrium in weighted networks.*

Rejecting the null hypothesis in favor of Hypothesis 1 opens the door to study which aspects of the network topology affect deviations from equilibrium. Our experimental design assigns subjects to nodes and networks randomly. This exogeneous variation addresses concerns of endogeneity, where the same individual characteristics affecting the position in a network can also affect choices.[23]

In particular, we focus on two aspects of the deviations: individual frequency and size. Comparing frequencies of deviations across networks is straightforward because, in every network, equilibrium predicts a frequency of zero deviations. In terms of size, we measure deviations by the Euclidian distance between the actual choices and the unique equilibrium predictions, $D_{\boldsymbol{a_i},\boldsymbol{a_i^*}} = \|\mathbf{a}_i - \mathbf{a}_i^*\| = \sqrt{\sum_{j=1}^{d_i}\left(a_{ij} - a_{ij}^*\right)^2}$. Notice that the highest possible deviation for participants of degree 2 and 3 are equal to $\sqrt{2}$, allowing deviations in both cases to be compared within and across networks because $D_{\boldsymbol{a_i},\boldsymbol{a_i^*}} \in \left[0, \sqrt{2}\right]$ for all treatments of the experiment.

With these comparable measures of deviations across networks, we can state more precise hypotheses about how the elements of the network structure affect behavior. In particular, our experimental design varies the degree of the participants, the diversity of types and the heterogeneity of the values of the conflicts. Thus, the three main hypotheses follow:

[23]For example, there exists evidence that biological traits as sex or levels of testosterone and cortisol affect position in networks (e.g. Ponzi et al., 2016; Kovářík et al., 2017) as well as preferences for competition (e.g. Gneezy and Rustichini, 2004; Niederle and Vesterlund, 2011). With naturally occurring data, one way to address this endogeneity problem is to introduce plausible instrumental variables (e.g., Acemoglu et al., 2015; König et al., 2017). However, even when endogeneity is addressed this way, a second concern is that the instrument might, nevertheless, fail to provide enough network topology variation to investigate its influence on behavior. While our experiment is restricted to four participant networks, the selection of our treatments provides almost exhaustive variation in the network topology of four participants.

**Hypothesis 2** (Deviations due to larger strategy space)**.** *An increase in degree leads to larger deviations from equilibrium play.*

**Hypothesis 3** (Deviations due to diversity of types)**.** *An increase in the diversity of types leads to larger deviations from equilibrium play.*

**Hypothesis 4** (Deviations due to heterogeneity of values)**.** *An increase in the heterogeneity of values leads to larger deviations from equilibrium play.*

# 5 Results

## 5.1 Data Overview

Our variable of interest is the vector of allocation shares, $\boldsymbol{a}_i$. Some features of the experimental design and the layout can potentially lead to order effects and framing effects, confounding this variable. To evaluate order effects, we test if the observed vectors of allocation shares depend on the position of the vertex assigned to each conflict in the decision interface. Results from Friedman tests suggest in every treatment that there are no order effects (see Appendix C for further details). To evaluate framing effects, the mean allocation shares of subjects assigned to the same position in the network is pooled; we test if the vector of mean allocation shares is the same across different positions, which are facing an isomorphic decision problem – for example, is the vector of average allocation shares by participants in position 1 in the complete network in the homogeneous the same as those in position 2, 3 and 4? The $p$-values of the associated Friedman tests are not significant at 5% in every case but two: complete and ring network in the heterogeneous treatment.[24]

Given this lack of evidence, we abstract from these potential confounds when reporting the experimental results throughout the paper. For each treatment, the summary statistics are presented in the sub-column "All Rounds" of Table 2 – the vector of sample mean allocations shares, $\bar{\boldsymbol{a}}_i = (\bar{a}_{i1}, \ldots, \bar{a}_{(i|N_i|)})$, the vector of the standard deviations for each allocation share, $(sd(a_{i1}), \ldots, sd(a_{(i|N_i|)}))$, and the number of observation. We also present these statistics for the first 10 rounds of each treatment, under the sub-column "Rounds 1-10".

[24] These tests include few observations and their negative results raise questions about power. Alternatively, for each isomorphic conflict in a network, one could test if the mean allocation share is the same across participants assigned to different positions in the network (1, 2, 3, 4) in a systematic manner. These tests include more observations but present a multiple hypotheses testing problem. The corresponding results show no significant effects in every conflict even if Bonferroni corrections are applied (see Appendix C for further details).

Table 2: Summary Statistics

| | | | Homogeneous | | Heterogeneous | |
|---|---|---|---|---|---|---|
| | | | Rounds 1-10 | All Rounds | Rounds 1-10 | All Rounds |
| Ring | | *Mean* | (0.519,0.480) | (0.508,0.492) | (0.680,0.319) | (0.697,0.303) |
| | | *sd* | $(0.212, 0.212)$ | $(0.216, 0.216)$ | $(0.225, 0.225)$ | $(0.227, 0.227)$ |
| | | *#Obs* | 320 | 640 | 320 | 640 |
| Complete | | *Mean* | (0.332,0.335,0.332) | (0.333,0.324,0.341) | (0.488,0.311,0.199) | (0.508,0.297,0.193) |
| | | *sd* | $(0.187, 0.168, 0.176)$ | $(0.174, 0.159, 0.177)$ | $(0.202, 0.139, 0.153)$ | $(0.210, 0.138, 0.147)$ |
| | | *#Obs* | 320 | 640 | 320 | 640 |
| Line | $i = \{2\}$ | *Mean* | $(0.547, 0.452)$ | $(0.574, 0.425)$ | (0.636,0.363) | (0.629,0.370) |
| | | *sd* | $(0.279, 0.279)$ | $(0.276, 0.276)$ | $(0.292, 0.292)$ | $(0.289, 0.289)$ |
| | | *#Obs* | 540 | 1080 | 270 | 540 |
| | $i = \{3\}$ | *Mean* | | | (0.617,0.382) | (0.612,00.387) |
| | | *sd* | | | $(0.266, 0.266)$ | $(0.261, 0.261)$ |
| | | *#Obs* | | | 270 | 540 |
| Diagonal | $i = \{1, 3\}$ | *Mean* | (0.475,0.524) | (0.487,0.512) | (0.592,0.407) | (0.591,0.408) |
| | | *sd* | (0.193, 0.193) | (0.189, 0.189) | (0.192, 0.192) | (0.193, 0.193) |
| | | *#Obs* | 320 | 640 | 320 | 640 |
| | $i = \{2, 4\}$ | *Mean* | (0.326,0.320,0.352) | (0.335,0.301,0.362) | (0.502,0.312,0.185) | (0.518,0.297,0.183) |
| | | *sd* | $(0.162, 0.163, 0.175)$ | $(0.158, 0.161, 0.179)$ | $(0.230, 0.178, 0.137)$ | $(0.221, 0.173, 0.135)$ |
| | | *#Obs* | 320 | 640 | 320 | 640 |

The unit of observation is the vector of allocation shares by participant $i$ in period $t$, $\boldsymbol{a}_i^t$.

The summary statistics show variations across the different treatments. For example, the introduction of the Heterogeneous values (such that $v_1 > v_2 > v_3$) shifts the participants' allocation shares in line with the value of the conflicts – i.e., $\bar{a}_{i1} > \bar{a}_{i2} > \bar{a}_{i3}$. In contrast, when comparing the entries in "Rounds 1-10" and the corresponding entries in "All Rounds", the mean allocation shares exhibit almost no variation within each treatment. This pattern in the data suggests that the change in participants' choices across time is exclusively due to incentives of the Heterogeneous treatment and not time trends (such as those expected if, for example, participants were learning).

To formally investigate potential time trends within a treatment, we compare the distributions of the within-subject average allocation shares for each conflict in the first and last 10 rounds. The associated $p$-values of the Wilcoxon signed rank tests are not significant at the 5% significance in every case.[25]

[25] While there is some evidence of learning in networks (e.g., Choi, 2012), multibattle contests with a Tullock contest success function in simpler experiments, without a network structure, also report a similar absence of time trends (see, for example, Dechenaux et al., 2015; Chowdhury et al., 2016). Similar results are obtained using a parametric analysis with the Dirichlet Covariate Model. Analysis is available upon request.

## 5.2 Equilibrium Test

For each network and degree, we estimate a Dirichlet Covariate Model, controlling for the heterogeneous treatment variable and session fix effects. In addition, given the lack of time trends, we focus on the within-subject average allocation share and, to account for participants only interacting within a specific network, we cluster the standard errors at the network level. Table 3 shows the vector of estimated mean allocation shares for each treatment, and the corresponding vector of estimated variances – a more detailed report of the Dirichlet Covariate Model results appears in Appendix E.1. In addition, Table 3 also includes the results of the non-linear Wald tests for the composite hypotheses associated to Table 1.

Table 3: Estimated Mean Allocation Shares and Variances

| | | Homogeneous | Heterogeneous |
|---|---|---|---|
| Ring | | (0.499,0.501)<br>(0.001,0.001) | (0.686,0.324)***<br>(0.009,0.009) |
| Complete | | (0.310,0.336,0.354)***<br>(0.003,0.003,0.003) | (0.510,0.316,0.174)<br>(0.013,0.015,0.009) |
| Line | $i=\{2\}$ | (0.51,0.49)**<br>(0.002,0.002) | (0.547,0.453)**<br>(0.009,0.009) |
| | $i=\{3\}$ | | (0.538,0.462)***<br>(0.005,0.005) |
| Diagonal | $i=\{1,3\}$ | (0.510,0.490)<br>(0.002,0.002) | (0.622,0.378)<br>(0.009,0.009) |
| | $i=\{2,4\}$ | (0.252,0.363,0.386)***<br>(0.002,0.002,0.002) | (0.366,0.368,0.266)***<br>(0.029,0.029,0.025) |

In a cell, the vector ($\hat{a}^*_{i1}$, $\hat{a}^*_{i2}$) and ($\hat{a}^*_{i1}$, $\hat{a}^*_{i2}$, $\hat{a}^*_{i3}$) corresponds to the estimated relative shares of degree 2 and 3 participants, respectively. We present the results of the non-linear Wald test using *** for p<0.01, ** for p<0.05 and * for p<0.1.

The unique interior equilibrium in pure strategies is rejected in 7 out of 11 instances at the 5% significance level.[26] At first sight, results in Table 3 suggest that Degree 3 treatments are more likely to present a rejection than Degree 2 treatments. Other treatments present less clear patterns. However, before presenting a more formal analysis about how the different treatments affect deviations from equilibrium, we explore the possibility that these rejections are mainly the result of few participants with extreme deviations – rather than widespread deviations.

[26] One could ask how the results from the Dirichlet Covariate Model, imposing a negative correlation across conflicts, compare with those from more popular methods, such as Ordinary Least Square (OLS), imposing independence across conflicts, instead. The results of the OLS are presented in Appendix E.2. Although the number of instances that we reject the equilibrium prediction is the same (i.e., 7), the pattern of rejections is completely different. In fact, the same conclusion is only reached in 3 instances.

For that, a similar Dirichlet Covariate Model is estimated for each individual separately.[27] Table 4 shows the percentage of participants, in each treatment, for which the equilibrium prediction is rejected at the 5% significance level. For example, the equilibrium prediction in the complete network is rejected for 21.8% of the participants in the Homogeneous treatment and 62.5% in the Heterogeneous treatment.

Table 4: Percentage of Rejections

| | | Homogeneous | Heterogeneous |
|---|---|---|---|
| Ring | | 3.1% | 71.8% |
| Complete | | 21.8% | 62.5% |
| Line | $i = \{2\}$ | 24% | 59.2% |
| | $i = \{3\}$ | | 44.4% |
| Diagonal | $i = \{1, 3\}$ | 31.2% | 37.5% |
| | $i = \{2, 4\}$ | 37.5% | 71.8% |

The percentage of participants for which the Nash equilibrium prediction is rejected indicates that deviations from equilibrium are a widespread phenomenon among our participants. In fact, the percentage is always higher than 20%, except in one treatment (the Ring network in the Homogeneous treatment). Furthermore, the percentage of rejection increases substantially when we compare a Homogeneous treatment with the corresponding Heterogeneous treatment. Other patterns are less clear. Results presented in Tables 3 and 4 can be summarized in the following result in relationship to Hypothesis 1.

**Result 1.** *There are significant and frequent deviations from the Nash equilibrium in weighted networks.*

[27] Only 5.24% of the observed decision in our experiment are non-interior allocations, which are spread across participants. Remember that the Dirichlet Covariate Model only accept interior allocations. Consequently, to deal with this problem, we adopt the second method described in Section 2. Similarly to Choi et al. (2007), an allocation $a_{ij} = 0$ is perturbed to $a_{ij} = \epsilon = 10^{-4}$, while the remaining allocations are reduced by $\frac{\epsilon}{\mathcal{D}-1}$. When $d_i > 2$ and we observe a decision such that $a_{ij} = 1$ and $a_{ik} = 0\ \forall k \neq j$, $a_{ij}$ is perturbed to $a_{ij} = 1 - \epsilon$ while the other allocations, $a_{ik}$ are increased by $\frac{\epsilon}{\mathcal{D}-1}$. This happens in 0.2% of our observations. In our case, the frequency of non-interior allocations is also informative about the extent to which risk aversion affects our results. It is plausible that risk averse participants select non-interior allocations in order to win a proper subset of conflicts. This effect should be more pronounced in the heterogeneous treatments where these participants would allocate resources to the highest valued conflicts. Only 56.29% of the non-interior allocations appear in a heterogeneous treatment; from this, 54.41% assign a zero allocation to the lowest valued conflict.

### 5.3 Sources of Deviations from Equilibrium

Result 1 establishes that the deviations from equilibrium reported in unweighted networks also extend to weighted networks. In addition, the combinations of treatments assigned to participants exogenously also permits an identification of the causal effect on those deviations by the elements of the structure of the network that our designed varies.

Formally, we measure deviations by the Euclidian distance $D_{\boldsymbol{a_i},\boldsymbol{a_i^*}}$. Then, we can estimate a Generalized Linear Square random effects model where $D_{\boldsymbol{a_i},\boldsymbol{a_i^*}}$ is regressed on a dummy for each of the treatments: Degree 3, Diverse, and Heterogeneous, taking value 1 if participant is assigned to a particular treatment and 0 otherwise; and the corresponding interactions between treatments. As before, we include session fix effects and cluster standard errors at the network level.

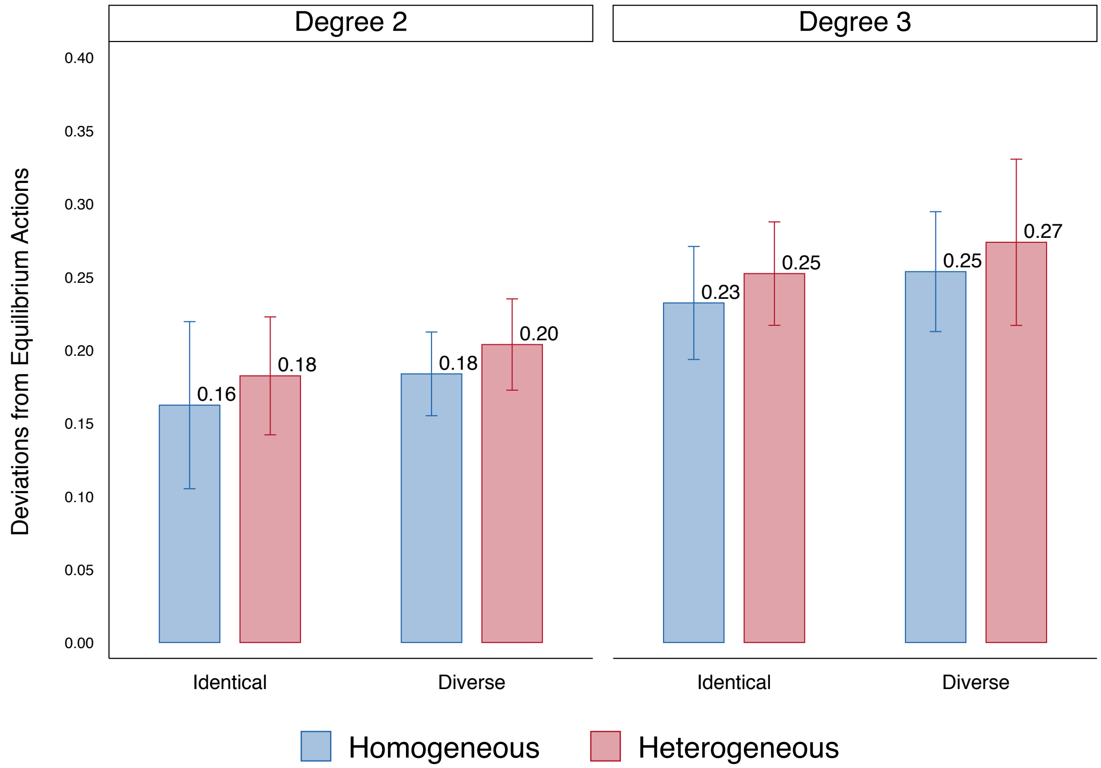
Figure 3 depicts the conditional estimated marginal effects of each treatment as well as the corresponding 95% confidence interval.[28] Thus, the results of the test of the remaining hypotheses can be visualized. First, the significance of the marginal effect of degree can be observed by comparing the bars in the left panel (Degree 2 treatments) and the corresponding bars in the right panel (Degree 3 treatments). Second, within each panel, the significance of the marginal effect of the diversity of degrees can be observed by comparing the left sub-panel (Identical treatments) and right sub-panel (Diverse treatments). Finally, within each sub-panel, the significance of the marginal effect of heterogeneity of values can be observed by comparing the blue bar (Homogeneous treatments) and the red bar (Heterogeneous treatments). If a bar is outside the confidence interval of the corresponding comparison, the marginal effect is significant at the 5% significance level.

Results in Figure 3 show that the only significant treatment is degree. When comparing Degree 2 and Degree 3 treatments, we observe a significant effect for all comparisons. More concretely, deviations from equilibrium increases by 0.07 in absolute terms, which, depending on the specific comparison, represents a relative increment between 35% and 43.75%. The Diverse treatment, and Heterogeneous treatment have the same relative effect (incrementing deviations between 8% to 12.5%). However, these effects are not statistically significant. Thus, we obtain the following results for the main hypotheses.

**Result 2.** *In line with Hypothesis 2, an increase in degree does lead to larger deviations from equilibrium play.*

[28] Full report of estimates appear in Appendix F.1.

Figure 3: Conditional Estimated Margins.




**Result 3.** *Contrary to Hypothesis 3, an increase in the diversity of types does not lead to larger deviations from equilibrium play.*

**Result 4.** *Contrary to Hypothesis 4, an increase in the heterogeneity of values does not lead to larger deviations from equilibrium play.*

# 6 Discussion

This paper proposes to use the Dirichlet Covariate model to empirically analyze the compositional data emerging from settings in which an individual needs to allocate some resources across different connections with other individuals. By using this method, point predictions of a growing literature of models with weighted networks and link-specific actions can now be tested, building on the current analysis of unweighted networks. This method is also able to recover and provide a meaningful interpretation of the estimated marginal effects – unlike the log-ratio transformation.

To illustrate this method, we report evidence from a novel experimental design implementing a variation of one of the aforementioned models (Franke and Öztürk, 2015). In this illustration, we present the results of testing the payoff maximizing equilibrium predictions – the results of other point predictions could also be obtained using the same method.[29] These results show that deviations from this equilibrium are significant and frequent in weighted networks (Result 1). This results extend current evidence in unweighted networks, which would appear impossible without our method.

From a behavioral point, one interesting aspect of our experiment is that, while the equilibrium prediction is unique, interior, and in pure strategies, avoiding some sources of deviations in simpler games, the equilibrium cannot be, in general, characterized using radicals. So, not only our participants but game theorists (and even computers) need to tolerate some deviation. Furthermore, the experimental design allows the effects of some important aspects of the network structure on the deviations from equilibrium to be studied. In particular, we find that a larger degree increases deviations from equilibrium significantly (Results 2-4). The structure of the network in our experiment is exogenous but these results have implications for endogenous network formation. Individuals need to decide to what extent an equilibrium with more connections compensates the deviations that would emerge by those additional connections. Variations in this trade-off could potentially explain reported deviations in experiments with endogenous network formation (see Choi et al., 2016; Kovářík et al., 2018).

[29] In fact, point predictions by heuristics are plausible in these settings. For example, participants could focus only on degree and split their budget equally among all conflicts. Another example, if participants recognize degree and the asymmetry of the values, they could allocate their budget in proportion to the value of the conflict with respect to the total sum of conflict values. Deviations from the point predictions are significant. Analysis is available upon request.

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# Appendix

# A Proofs

## A.1 Existence of an unique interior Nash Equilibrium in pure strategies

This proof has two steps. First, we establish that our game satisfy the sufficient conditions for a unique equilibrium in pure strategies. This first step follows Rosen (1965) and Goodman (1980) and requires an interior equilibrium to be assumed. Second, we provide sufficient conditions for the equilibrium of the game to be interior.

**Step 1**: Theorems 1 and 2 in Rosen (1965) prove that, for a unique equilibrium in pure strategies to exist in a $N$-person concave game, it is sufficient that the joint payoff function $\sigma(\boldsymbol{a}, \boldsymbol{r}) = \sum_{i \in N} r_i \pi_i$, where $\boldsymbol{a} = (\boldsymbol{a_1}, ..., \boldsymbol{a_N})$ and $\boldsymbol{r} = (\boldsymbol{r_1}, ..., \boldsymbol{r_N})$, is *diagonally strictly concave.* However, before defining the property of diagonally strictly concave, in order to apply these theorems, the payoff function of every player $i$, $\pi_i$ and the strategy set of each player $i$, $\boldsymbol{a_i}$, needs to satisfy certain properties. It is immediate that, in our game, the strategy set of each player $\boldsymbol{a_i}$ is convex and compact; and the strategy sets of the players are orthogonal. Also, in our game, the payoff function of player $i$ is concave in his own strategy (see Claim 1 below); but not continuous. More specifically, this payoff function has a discontinuity – i.e., whenever $a_{ij} = a_{ji} = 0$, the contest success function "jump" to $\frac{1}{2}$ and, consequently, the payoff function of player $i$ is dicontinuous at that point. To circumvent this problem, we modify the Tullock contest success function following Myerson and Wärneryd (2006):

$$\tilde{p}_{ij} = \frac{a_{ij} + \delta}{a_{ij} + a_{ji} + 2\delta}.$$

The resulting payoff function of player $i$ including $\tilde{p}_{ij}$ is continuous; in the limit $\delta \to 0$, this function $\tilde{p}_{ij}$ coincides with $p_{ij}$ at every point

$$\lim_{\delta \to 0} \tilde{p}_{ij} = \lim_{\delta \to 0} \frac{a_{ij} + \delta}{a_{ij} + a_{ji} + 2\delta} = p_{ij}$$

Thus, in the limit, when $\delta \to 0$, the first order conditions and equilibrium results should be the same for games using either contest success functions, $p_{ij}$ or $\tilde{p}_{ij}$, and their corresponding payoff functions. Once continuity is satisfied with the transformed game, regular properties of differentiability are also met.

In addition to these properties satisfied by our modified game using $\tilde{p}_{ij}$, Rosen (1965) restrict his attention to strictly interior equilibrium (p.523). Goodman (1980) shows the property diagonally strictly concavity is implied by the following three properties:

i) $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ is strictly concave in $\mathbf{a_i}$ for all $\mathbf{a_{-i}}$,

ii) $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ is convex in $\mathbf{a_{-i}}$ for all $\mathbf{a_i}$,

iii) $\rho(\mathbf{a}, \mathbf{r}) = \sum_{i \in N} r_i \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ is concave in $\mathbf{a_i}$ for some $\mathbf{r}$ with $r_i > 0$ for all $i \in N$;

We verify that these properties are satisfied in our modified game:

*Claim 1:* $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G}))$ *is strictly concave in* $\mathbf{a_i}$ *for all* $\mathbf{a_{-i}}$*.*

Consider the Hessian matrix of $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ with respect to player $i$'s profile strategy. In the diagonal of this matrix we have $\dfrac{\partial^2 \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})}{\partial a_{ij}^2} = v_{ij} \dfrac{-2a_{ji}}{(a_{ij} + a_{ji})^3} < 0$ and in the off-diagonal entries we have $\dfrac{\partial^2 \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})}{\partial a_{ij} \partial a_{ik}} = 0$ for every $k \neq j \in N_i$ due to the *mutually statistically independence* between conflicts. Therefore, the Hessian matrix is negative definite and, consequently, $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ is strictly concave in $\mathbf{a_i}$ for all $\mathbf{a_{-i}}$.

*Claim 2:* $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ *is convex in* $\mathbf{a_{-i}}$ *for all* $\mathbf{a_i}$*.*

Consider the Hessian matrix of $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ with respect to player $j$'s profile strategy. In the diagonal of this matrix we have $\dfrac{\partial^2 \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})}{\partial a_{ji}^2} = v_{ij} \dfrac{2a_{ij}}{(a_{ij} + a_{ji})^3} > 0$ or $\dfrac{\partial^2 \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})}{\partial a_{ki}^2} = 0$ and in the off-diagonal entries we have $\dfrac{\partial^2 \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})}{\partial a_{ji} \partial a_{ki}} = 0$, for all $k \neq j \in N$. This implies that the Hessian matrix of $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ respect to player $j$'s action is positive semi-definite and, consequently, $\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ is convex in $\mathbf{a_{-i}}$ for all $\mathbf{a_i}$.

*Claim 3:* $\rho(\mathbf{a}, \mathbf{r}) = \sum_{i \in N} r_i \pi_i(\mathbf{a_i}, \mathbf{a_{-i}}, \mathcal{G})$ *is concave in* $\mathbf{a_i}$ *for some* $\mathbf{r}$ *with* $r_i > 0$ *for all* $i \in N$*.*

In our modified game, $\rho(\mathbf{a}, \mathbf{r}) = \sum_{i \in N} r_i \sum_{j \in N_i} v_{ij} \frac{a_{ij}+\delta}{a_{ij}+a_{ji}+2\delta}$. Consider the same $r_i$ for each player, then $\rho(\mathbf{a}, \mathbf{r}) = r \sum_{i \in N} \sum_{j \in N_i} v_{ij}\, \tilde{p}_{ij}$. As $\tilde{p}_{ik} = 1 - \tilde{p}_{ki}$, the double summation will translate into a set of summation of the type $v_{ij}\tilde{p}_{ij} + v_{ji}\tilde{p}_{ji} = v_{ij}\ \forall (ij) \in B$. Therefore, for $r_i = r$ for all $i \in N$, $\rho(\mathbf{a}, \mathbf{r}) = r \sum_{(ij) \in B} v_{ij}$ so $\rho(\mathbf{a}, \mathbf{r})$ is a constant and, consequently, concave in $\mathbf{a_i}$.

Thus, if the optimal is interior, then, there exists a unique equilibrium in pure strategies.

**Step 2**: By contradiction, we show that $\frac{v_{ih}}{v_{il}} > \frac{1}{4}$ is sufficient for the optimal to be interior ($a_{ij} > 0 \ \forall i \in N, j \in N_i$).

Assume that if $\frac{v_{ih}}{v_{il}} > \frac{1}{4}$, then, for player $i$, $a^*_{ij} = 0$ for conflict $(ij)$ and $a^*_{ik} > 0 \ \forall k \neq j$. We show this extreme case but it is immediate that the same argument and conditions hold for every set of players and conflicts such that their optimal allocation is assumed to be 0.

In the assumed equilibrium, it should be the case that

$$\frac{\partial \pi(\boldsymbol{a}_i, \boldsymbol{a}_{-i}, G)}{\partial a_{ih}} = \frac{\partial \pi(\boldsymbol{a}_i, \boldsymbol{a}_{-i}, G)}{\partial a_{ik}} > \frac{\partial \pi(\boldsymbol{a}_i, \boldsymbol{a}_{-i}, G)}{\partial a_{ij}} \ \forall k \neq j.$$

For now, we focus on the first and last term and, taking into account the assumption that $a_{ij} = 0$, we get

$$\frac{v_{ih} a_{hi}}{(a_{ih} + a_{hi})^2} > \frac{v_{ij}}{a_{ji}} \Rightarrow \frac{a_{ji} a_{hi}}{(a_{ih} + a_{hi})^2} > \frac{v_{ij}}{v_{ih}}.$$

Notice that the assumption $\dfrac{v_{il}}{v_{ih}} > \dfrac{1}{4}$ implies $\dfrac{v_{ij}}{v_{ih}} > \dfrac{1}{4}$. Therefore,

$$\frac{a_{ji} a_{hi}}{(a_{ih} + a_{hi})^2} > \frac{1}{4}. \tag{3}$$

When $a^*_{ij} = 0$, player $j$ has an incentive to choose $a^*_{ji} = \epsilon$ to win conflict $(ij)$ with $\lim_{\delta \to 0} \tilde{p}_{ji} = 1$ while reducing only marginally the probability of winning any conflict $(jk) \in B$, where $k \neq i$. $\epsilon$ denotes usually an arbitrarily small positive number. In our modified game, $a_{jk}$ can also be arbitrarily small when player $j$ is spreading his budget across conflicts $\frac{1}{|N_j|}$, and the number of conflicts of player $j$, $\mid N_j \mid$, is arbitrarily large. To avoid problems arising from the ambiguity of the term arbitrarily, we bound $\epsilon < \frac{1}{|N_j|} \ \forall j$ and note that, for player $j$, the incentive to choose $a^*_{ji} = \epsilon < \frac{1}{|N_j|}$ still holds for any arbitrarily large number of conflicts.

By substituting $a_{ji} = \epsilon$ in expression (3), we obtain

$$\frac{\epsilon a_{hi}}{(a_{ih} + a_{hi})^2} > \frac{1}{4}. \tag{4}$$

The expression in the left hand side of this inequality reaches the maximum when $a_{ih} = a_{hi}$. Thus, we evaluate expression (4) at the point $a_{ih} = a_{hi}$:

$$\frac{\epsilon a_{hi}}{(a_{ih}+a_{hi})^2} > \frac{1}{4} \Leftrightarrow \frac{\epsilon a_{hi}}{(2a_{hi})^2} > \frac{1}{4} \Leftrightarrow \frac{\epsilon}{a_{hi}} > 1 \Leftrightarrow \epsilon > a_{hi} = a_{ih}.$$

Recall that the marginal payoffs for conflicts that receive a strictly positive allocation are equal and battlefields are order such that $v_{ih} > v_{ik}\ \forall k \in N_i$. Thus, in equilibrium, $a_{ih} \geq a_{ik} > a_{ij} = 0; \forall k \neq j$, which implies that the allocation $a_{ih}$ is higher than the average (i.e., $a_{ih} > \frac{1}{|N_i - 1|}$). Hence, $\epsilon > a_{ih} > \frac{1}{|N_i-1|} > \frac{1}{|N_i|}$ Thus, we have reached a contradiction. $\square$

## A.2 Equilibrium Characterization Impossibility/Possibility

The first proof of this section substantiates Proposition 2 and proceeds in three steps. First, we show that the equilibrium allocation shares of any four connected players along a path are related through the first order conditions. Second, we show that, in this case, even when we take the equilibrium allocation shares of the other players as given, the resulting expression is a general polynomial of degree 6. Longer paths lead to general polynomials of higher degree. Finally, using the Abel-Ruffini Theorem, which is valid for general polynomials, we finish the proof of proposition 2.

**Step 1**: This step follows from the first order conditions. When Proposition 1 holds, the Lagrange function for each player $i$ is given by

$$\mathcal{L} = \sum_{j \in N_i} \frac{v_{ij} a_{ij}}{a_{ij} + a_{ji}} + \lambda_i (1 - \sum_{j \in N_i} a_{ij}), \quad \forall i \in N \text{ and } \forall (ij) \in B$$

and the first order conditions for every player $i \in N$ are

$$\frac{\partial \mathcal{L}}{\partial a_{ij}} = \frac{v_{ij} a_{ji}}{(a_{ij} + a_{ji})^2} - \lambda_i = 0, \quad \forall j \in N_i \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = (1 - \sum_{j \in N_i} a_{ij}) = 0 \ \text{ (budget constraint)} \tag{6}$$

By rearranging expression 5, we obtain

$$\frac{v_{ij} a_{ji}}{(a_{ij} + a_{ji})^2} = \lambda_i, \ \forall i \in N \text{ and } \forall (ij) \in B. \tag{7}$$

Thus, our problem is a system of $(n + \sum_{i \in N} d_i)$ non-linear equations with $(\sum_{i \in N} d_i)$ unknowns.

Furthermore, for any two players $q$ and $k$ in the set $N_i$, expression (7) implies that

$$\frac{v_{iq}a_{qi}}{(a_{iq}+a_{qi})^2}=\lambda_i \text{ and } \frac{v_{ik}a_{ki}}{(a_{ik}+a_{ki})^2}=\lambda_i.$$

As the left hand side in both expressions are equal to $\lambda_i$, we equate them and rearrange such that

$$a_{qi}=a_{ki}\frac{v_{ik}}{v_{iq}}\frac{(a_{iq}+a_{qi})^2}{(a_{ik}+a_{ki})^2}. \tag{8}$$

In the underlying undirected network, there is a path $\mathcal{P}=\{qi,ik\}$ of length 2 between players $q$ and $k$. Expression (8) shows that the optimal strategy of player $q\in N_i$ $(a_{qi})$ depends not only on player $i$'s choice against him $(a_{iq})$ but also on the choice of player $i$ against $k$, which is connected to $i$ but not $q$, $(a_{ik})$, and the choice of player $k$ against $i$ $(a_{ki})$. This relation is quadratic.

More generally, the relation of the strategies of players that belongs to a path in the network can be represented as follows. Consider an arbitrary path $\mathcal{P}=\{i_1i_2,i_2i_3,\ldots,i_{j-1}i_j\}$ in the network and the allocation associated to the first conflict of that path

$$a_{i_1i_2}=a_{i_2i_3}\frac{v_{i_2i_3}}{v_{i_2i_1}}\frac{(a_{i_1i_2}+a_{i_2i_1})^2}{(a_{i_2i_3}+a_{i_3i_2})^2}. \tag{9}$$

This expression (9) follows the same structure as expression (8), which can be used recurrently and substitute $a_{i_2i_3}$ in expression (9) for the corresponding term:

$$a_{i_1i_2}=\left(a_{i_3i_4}\frac{v_{i_3i_4}}{v_{i_3i_2}}\frac{(a_{i_2i_3}+a_{i_3i_2})^2}{(a_{i_3i_4}+a_{i_4i_3})^2}\right)\frac{v_{i_2i_3}}{v_{i_2i_1}}\frac{(a_{i_1i_2}+a_{i_2i_1})^2}{(\left(a_{i_3i_4}\frac{v_{i_3i_4}}{v_{i_3i_2}}\frac{(a_{i_2i_3}+a_{i_3i_2})^2}{(a_{i_3i_4}+a_{i_4i_3})^2}\right)+a_{i_3i_2})^2}. \tag{10}$$

Expression (10) refers to conflict $a_{i_1i_2}$ but we have not include yet the first order conditions of player $i_1$,

$$\frac{\partial\mathcal{L}_{i_1}}{\partial a_{i_1i_2}}=\frac{a_{i_2i_1}}{(a_{i_1i_2}+a_{i_2i_1})^2}-\lambda_{i_1}=0.$$

By replacing $a_{i_1i_2}$ with the expression defined in equation 10, we get

$$\frac{v_{i_2i_1}a_{i_2i_1}}{(\left(a_{i_3i_4}\frac{v_{i_3i_4}}{v_{i_3i_2}}\frac{(a_{i_2i_3}+a_{i_3i_2})^2}{(a_{i_3i_4}+a_{i_4i_3})^2}\right)\frac{v_{i_2i_3}}{v_{i_2i_1}}\frac{(a_{i_1i_2}+a_{i_2i_1})^2}{(\left(a_{i_3i_4}\frac{v_{i_3i_4}}{v_{i_3i_2}}\frac{(a_{i_2i_3}+a_{i_3i_2})^2}{(a_{i_3i_4}+a_{i_4i_3})^2}\right)+a_{i_3i_2})^2}+a_{i_2i_1})^2}-\lambda_{i_1}=0 \tag{11}$$

For the purpose of our proof is enough that we stop the recurrent substitution in a path of length 3. However, one could continue with the recurrent substitutions along the path, reaching expressions of higher order.

**Step 2**: Let's assume the most favorable case, where all unknowns in expression (11) are known except one, $a_{i_4 i_3}$. To simplify notation, let $a_{i_4 i_3} = x$ and every other combination of parameters will be assumed to be known and therefore substituted by a known constant. Thus, we can rewrite expression 11 as

$$\frac{c_1}{\left(\frac{c_2}{(c_3+x)^2}\frac{c_4}{(\frac{c_5}{(c_3+x)^2}+c_6)^2}+c_7\right)^2} - c_0 = 0$$

If we expand this expression and reorganize it

$$\frac{c_1(c_5+c_6(c_3+x)^2)^4}{(c_2c_4(c_3+x)^2+c_7(c_5+c_6(c_3+x)^2)^4} - c_0 = 0$$

$$c_0(c_2c_4(c_3+x)^2+c_7(c_5+c_6(c_3+x)^2)^4 - c_1(c_5+c_6(c_3+x)^2)^4 = 0.$$

The final expression is the following general polynomial of degree 6, which is irreducible in $\mathcal{C}$, and, consequently in $\mathcal{R}$.

$$c_8x^8+c_9x^7+c_{10}x^6+c_{11}x^5+c_{12}x^4+c_{13}x^3+c_{14}x^2+c_{15}x+c_{16}=0.$$

This expression applies to any arbitrary path of length three in the network of the game (provided that it exists). In general, we can continue with a recurrent substitution of terms along any path in the network. Denote the length of a path between two players by $L$. Then, due to the quadratic nature of expression (7), the resulting expression is also a general polynomial,

$$c_{2^L}x^{2^L}+c_{2^L-1}x^{2^L-1}+c_{2^L-2}x^{2^L-2}+\ldots+c_2x^2+c_1x+c_0=0.$$

So, even when have solved all other unknowns, we still have to find the solution to a general polynomyal of high degree. When the path is three, this general polynomial is of degree 8.

**Step 3**: For the last step, we refer to the Abel-Ruffini Theorem:

**Theorem 1.** *Abel-Ruffini Theorem*

*A general algebraic equation of degree $\geq 5$ cannot be solved in radicals alone.*

The proof is valid for general polynomials, which are irreducible. Rosen (1995) provides details of the proof and a historical account of the contribution in relationship to Galois theory. $\square$

The next proof substantiates the claim that under some conditions of symmetry a solution can be expressed using radicals. More concretely, as stated in the main text, we shall show that Claim 1 below is true.

*Claim 1: In a weighted q-regular where $\sum_{j\in N_i} v_{ij} = \bar{V}$ for every $i \in N$. In this case, the equilibrium strategies for every player $i$ over each conflict $(ij) \in B$ is $a^*_{ij} = \frac{v_{ij}}{\bar{V}}$.*

Proposition 1 establishes that the solution is unique. So, we just check that the proposed equilibrium satisfy the first order conditions. It is straightforward to see that the proposed equilibrium satisfies the budget constraint.

Recall expression (7). Dividing both sides by $\lambda_i$, summing over element $k \in N_i$, and rearranging, we obtain

$$\frac{\sum_{k\in N_i}\sqrt{v_{ik}a_{ki}}}{\sum_{k\in N_i}(a_{ik}+a_{ki})} = \sqrt{\lambda_i}, \;\; \forall i \in N \;\text{ and }\; \forall (ij) \in B. \tag{12}$$

Expression (7) can be rearranged in a similar manner

$$\frac{\sqrt{v_{ij}a_{ji}}}{(a_{ij}+a_{ji})} = \sqrt{\lambda_i}, \;\; \forall i \in N \;\text{ and }\; \forall (ij) \in B. \tag{13}$$

By equating expressions (12) and (13) and rearranging, we obtained the best response function of player $i$ against player $j$,

$$a_{ij} = \frac{\sqrt{v_{ij}a_{ji}}}{\sum_{k\in N_i}\sqrt{v_{ik}a_{ki}}}\sum_{k\in N_i}(a_{ik}+a_{ki}) - a_{ji} \tag{14}$$

According to Claim 1, in equilibrium, $a^*_{ij}$ should be $\frac{v_{ij}}{\bar{V}}$ when all other allocations are in equilibrium. Thus, we can check this claim through the best-response above:

$$a_{ij} = \frac{\sqrt{v_{ij}\frac{v_{ji}}{\bar{V}}}}{\sum_{k\in N_i}\sqrt{v_{ik}\frac{v_{ki}}{\bar{V}}}}\sum_{k\in N_i}\left(\frac{v_{ik}}{\bar{V}}+\frac{v_{ki}}{\bar{V}}\right) - \frac{v_{ji}}{\bar{V}}$$

Using the fact that $v_{ij} = v_{ji}$ and $\sum_{k \in N_i} v_{ik} = \bar{V}$, we simplify

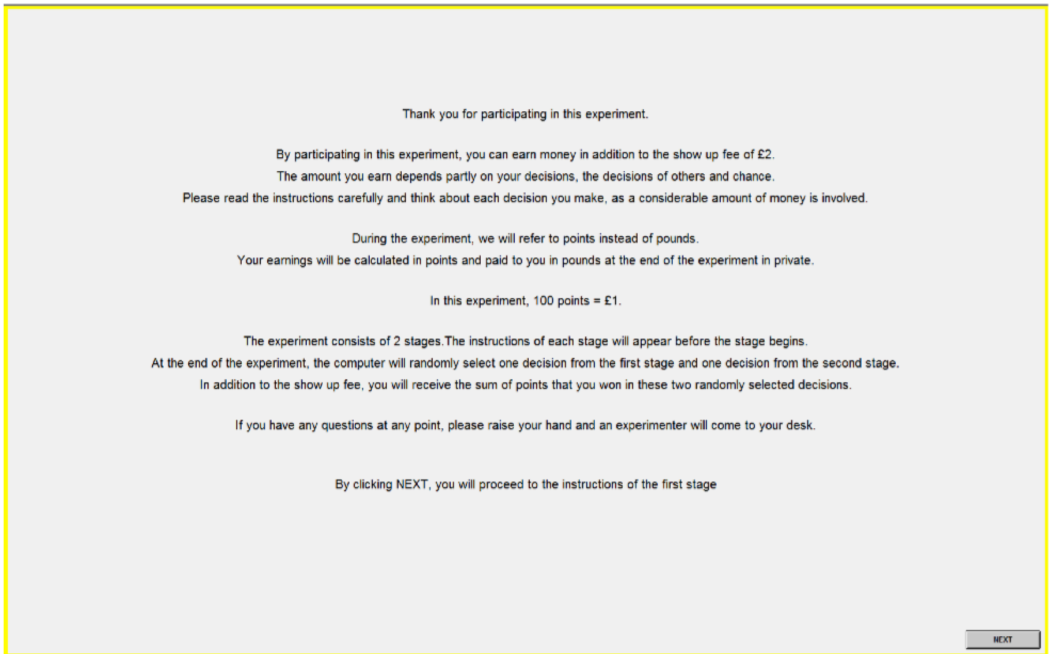
$$a_{ij} = \frac{\sqrt{v_{ij}\frac{v_{ij}}{\bar{V}}}}{\sum_{k \in N_i} \sqrt{v_{ik}\frac{v_{ik}}{\bar{V}}}} \sum_{k \in N_i} (\frac{v_{ik}}{\bar{V}} + \frac{v_{ik}}{\bar{V}}) - \frac{v_{ij}}{\bar{V}} \Leftrightarrow a_{ij} = \frac{\frac{v_{ij}}{\sqrt{\bar{V}}}}{\sum_{k \in N_i} \frac{v_{ik}}{\sqrt{\bar{V}}}} \sum_{k \in N_i} (2\frac{v_{ik}}{\bar{V}}) - \frac{v_{ij}}{\bar{V}}$$

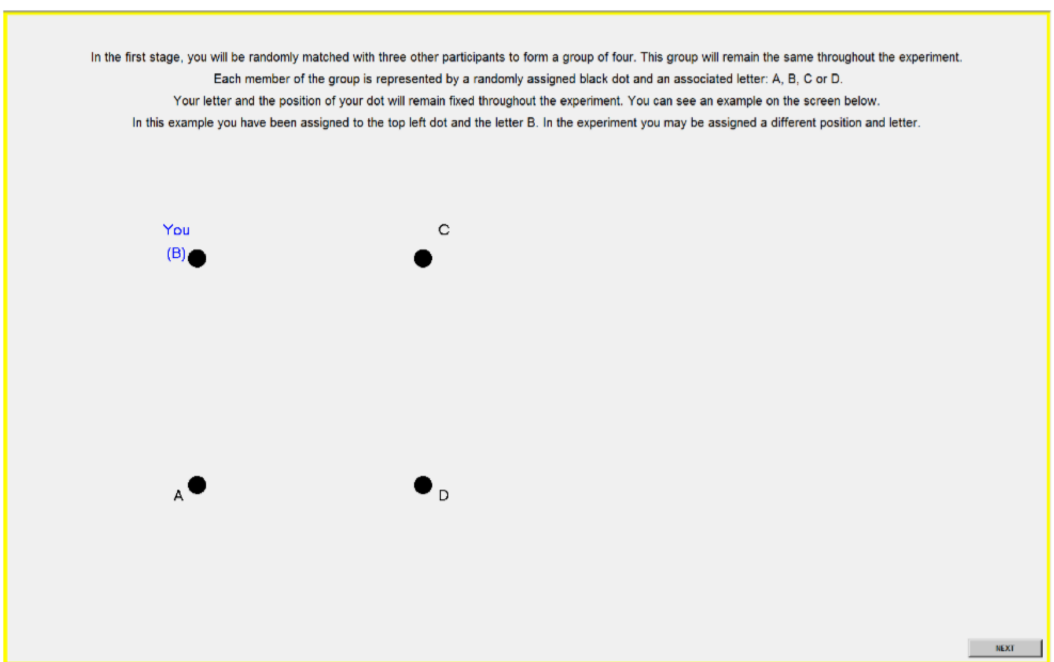
$$\Leftrightarrow a_{ij} = \frac{\frac{v_{ij}}{\sqrt{\bar{V}}}}{\frac{\sum_{k \in N_i} v_{ik}}{\sqrt{\bar{V}}}} \frac{2}{\bar{V}} \sum_{k \in N_i} v_{ik} - \frac{v_{ij}}{\bar{V}} \Leftrightarrow a_{ij} = \frac{2v_{ij}}{\bar{V}} - \frac{v_{ij}}{\bar{V}} = \frac{v_{ij}}{\bar{V}}$$

$\square$

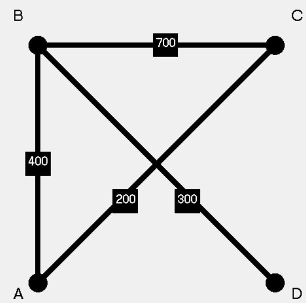
# B Instructions and Screenshot of the graphical representation

## B.1 Instructions



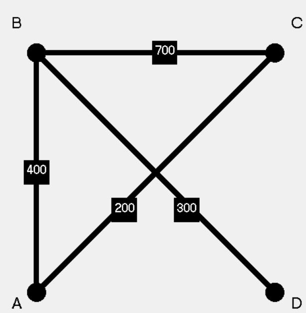
In the first stage, you will face 20 decision problems.
In each decision problem, every participant will be connected to either one, two or three boxes by a line between their dots and the box.
Two participants are always connected to one box. Each box contains a particular number of points, which may vary across boxes.
You can see an example on the screen below. The connections and the boxes may be different in the experiment.




In this example:
Participants A and B are connected to a box that contains 400 points.
Participants B and C are connected to a box that contains 700 points.
Participants A and C are connected to a box that contains 200 points.
Participants B and D are connected to a box that contains 300 points.
Thus,
Participant B is connected to three boxes.
Participants A and C are connected to two boxes.
Participant D is connected only to one box.

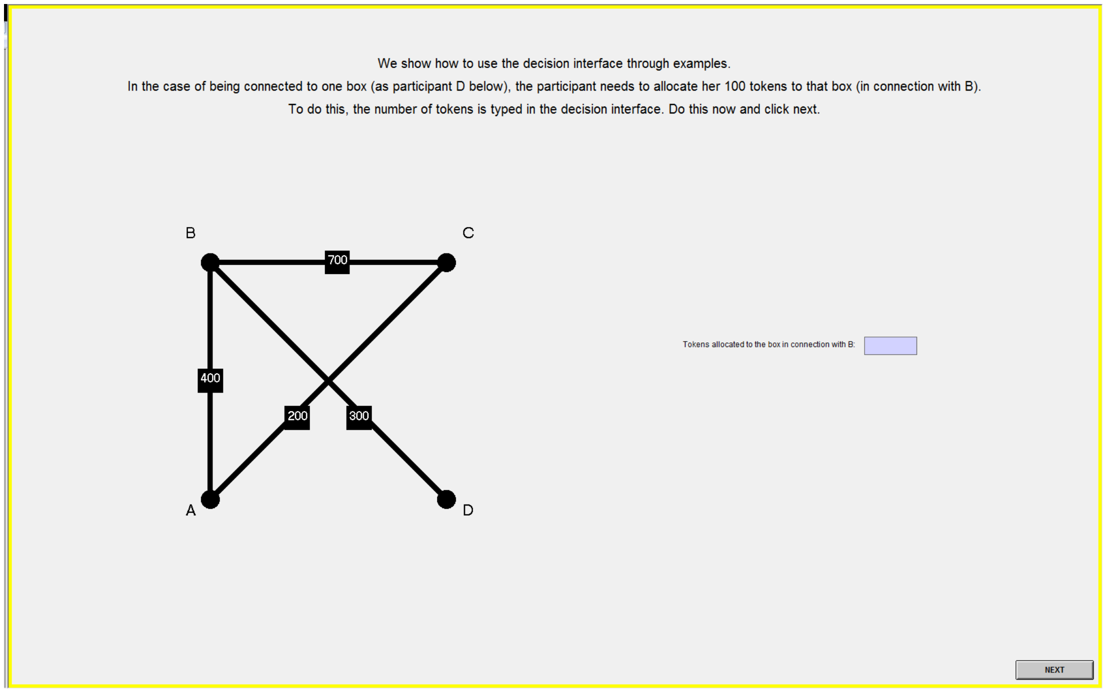
NEXT

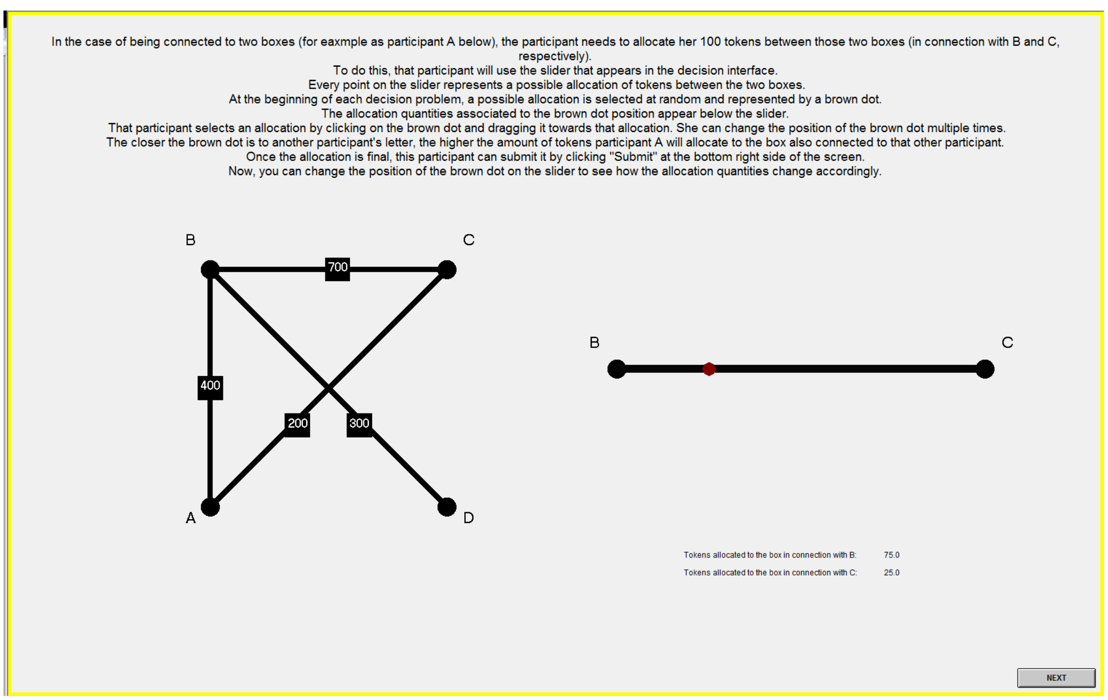
In each decision problem, the objective is to win as many points from the boxes as possible.
To that aim, each participant receives 100 tokens and is required to allocate all these tokens between the boxes that she is connected with.
An allocation is selected using the right hand side panel on your screen (decision interface).
As participants will be connected to either one, two or three boxes, it will be explained next how the decision interface looks in each of these cases.




Decision Interface

NEXT


We show how to use the decision interface through examples.
In the case of being connected to one box (as participant D below), the participant needs to allocate her 100 tokens to that box (in connection with B).
To do this, the number of tokens is typed in the decision interface. Do this now and click next.
B
C
700
400
200
300
A
D
Tokens allocated to the box in connection with B:
NEXT

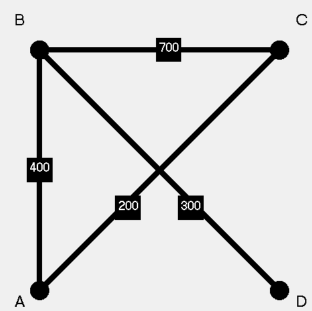

In the case of being connected to two boxes (for eaxmple as participant A below), the participant needs to allocate her 100 tokens between those two boxes (in connection with B and C, respectively).
To do this, that participant will use the slider that appears in the decision interface.
Every point on the slider represents a possible allocation of tokens between the two boxes.
At the beginning of each decision problem, a possible allocation is selected at random and represented by a brown dot.
The allocation quantities associated to the brown dot position appear below the slider.
That participant selects an allocation by clicking on the brown dot and dragging it towards that allocation. She can change the position of the brown dot multiple times.
The closer the brown dot is to another participant's letter, the higher the amount of tokens participant A will allocate to the box also connected to that other participant.
Once the allocation is final, this participant can submit it by clicking "Submit" at the bottom right side of the screen.
Now, you can change the position of the brown dot on the slider to see how the allocation quantities change accordingly.
B
C
700
400
200
300
A
D
B
C
Tokens allocated to the box in connection with B: 75.0
Tokens allocated to the box in connection with C: 25.0
NEXT

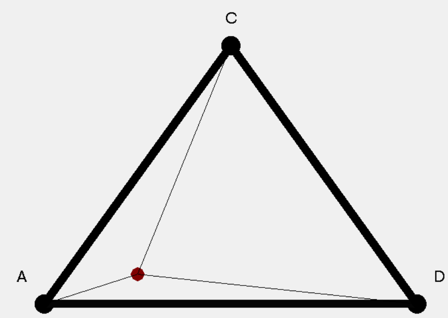
In the case of being connected to three boxes (as participant B below), the participant needs to allocate her 100 tokens between those three boxes (in connection with A, C and D, respectively). To do this, that participant will use the triangle that appears in the decision interface.

Each point inside the triangle represents a possible allocation of tokens between the three boxes.
At the beginning of each decision problem, a possible allocation is selected at random and represented by a brown dot.
The allocation quantities associated to the brown dot position appear below the triangle.

That participant selects an allocation by clicking on the brown dot and dragging it towards that allocation. She can change the position of the brown dot multiple times.
The closer the brown dot is to another participant's letter, the higher the amount of tokens participant B will allocate to the box also connected to that other participant.
Once the allocation is final, this participant can submit it by clicking "Submit" at the bottom right side of the screen.

Now, you can change the position of the brown dot inside the triangle to see how the allocation quantities change accordingly.




Tokens allocated to the box in connection with A: 77.4
Tokens allocated to the box in connection with C: 10.4
Tokens allocated to the box in connection with D: 12.2

NEXT

## How do you win the points of a box?

Your chances of winning the points of a box are given by the number of tokens that you allocated to that box divided by the total number of tokens allocated by you and the other participant connected to that box.

In the case that both participants allocate zero tokens to the same box, each participant has 50% chance of winning the points in that box.

After every participant has made their decision, for each box, you will be shown your chances of winning that box as the Blue area of a rectangle. You will be also shown the chances of the other participant to win that box as a Red area. The size of each area is proportional to the respective chances of winning. This is shown on the screen below.

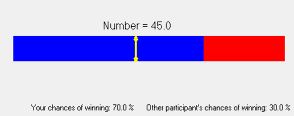
In the rectangle, the computer will draw a number between 0 and 100 (0.1 decimal allowed) and show a Yellow line, accordingly. This line will determine the winner of that box.

If the Yellow line is inside the **Blue area**, you **win** that box and the associated points will be yours.

If the Yellow line is inside the **Red area**, you **lose** that box and you will get no points.

The winner of each box is determined independently.

For example




NEXT

## Example

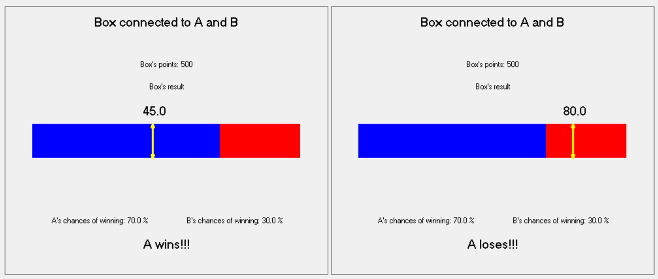
Imagine that, participant A and B are connected to a box that contains 500 points.

Participant A has a 70% chance of winning the points of the box and participant B has a 30% chance of winning them.

The Blue area presents A's chances of winning (70% of the rectangle) while the Red area represents B's chances of winning (30% of the rectangle).

In the left panel, the computer draws a number randomly and, as result of this, the Yellow line appears at 45. Because 45 is inside the Blue area (A's chances of winning), participant A wins the 500 points.

In the right panel, the computer draws a number randomly and, as result of this, the Yellow line appears at 80. Because 80 is inside the Red area (B's chances of winning), participant A loses the box and gets no points.




NEXT

## Summary

The experiment consists of 2 stages, you will face 20 decision problems in each stage.

You will be matched with three other participants to form a group of four. Each member of the group is represented by a black dot and an associated letter. Each member of the group is going to be connected to some boxes which contains a particular number of points.

In every decision problem, your objective is to win as many points from the boxes as possible. To that aim, you are asked to allocate 100 tokens between the boxes that you are connected with.

Your chances of winning the points of a given box are given by the number of tokens that you have allocated to that box divided by the total number of tokens allocated by you the other participant connected to that box. In the case that both participants allocate zero tokens to the same box, each participant has 50% chance of winning the points of that box.

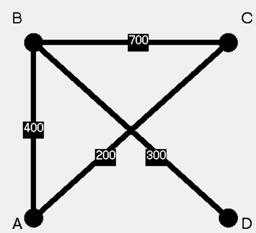
After every participant has chosen a final allocation, the computer will draw a number between 0 and 100 for each box independently. This number will determine who the winner of each box is. The winner will receive the points associated to that box.

In addition to the £2 show up fee, you will be paid according to the sum of the points that you won in two selected decision problems.

Remember: 100 points = £1.

By clicking the START button, you consent to participate in this experiment. Even if you decide to take part in the experiment, you are free to withdraw at any time. Withdrawing from this particular experiment will not affect your relation with the laboratory.

Start



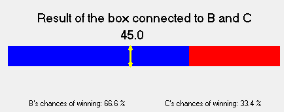

In this network:

- Participants A and B are connected to a box that contains 400 points. Participant B needs to allocate 100 tokens between 2 boxes.
- Participants B and C are connected to a box that contains 300 points. Participant B needs to allocate 100 tokens between 3 boxes.
- Participants B and D are connected to a box that contains 300 points. Participant D needs to allocate 100 tokens between only 1 boxes.
- Participants A and C are connected to a box that contains 700 points. Participant A needs to allocate 100 tokens between 2 boxes.

In the example above, if participant B allocates 50 tokens and participant C allocates 25 tokens to win the points of the box connected between them.

Then,

- Participant B chances of winning 700 points are 50%.
- Participant B chances of winning 200 points are 66.6% [ 50/(50+25) ].
- Participant B chances of winning 700 points are 33.3% [ 25/(50+25) ].
- Participant B chances of winning 700 points are 66.6% [ 50/(50+25) ].




Then,

- Participant B chances of winning are 66.6%, the box realization is 30 and B loses.
- Participant B chances of winning are 66.6%, the box realization is 45 and B wins.
- Participant C chances of winning are 33.4%, the box realization is 66.6 and C wins.
- Participant C chances of winning are 33.4%, the box realization is 45 and C wins.

OK

## B.2 Screenshot of the graphical representation

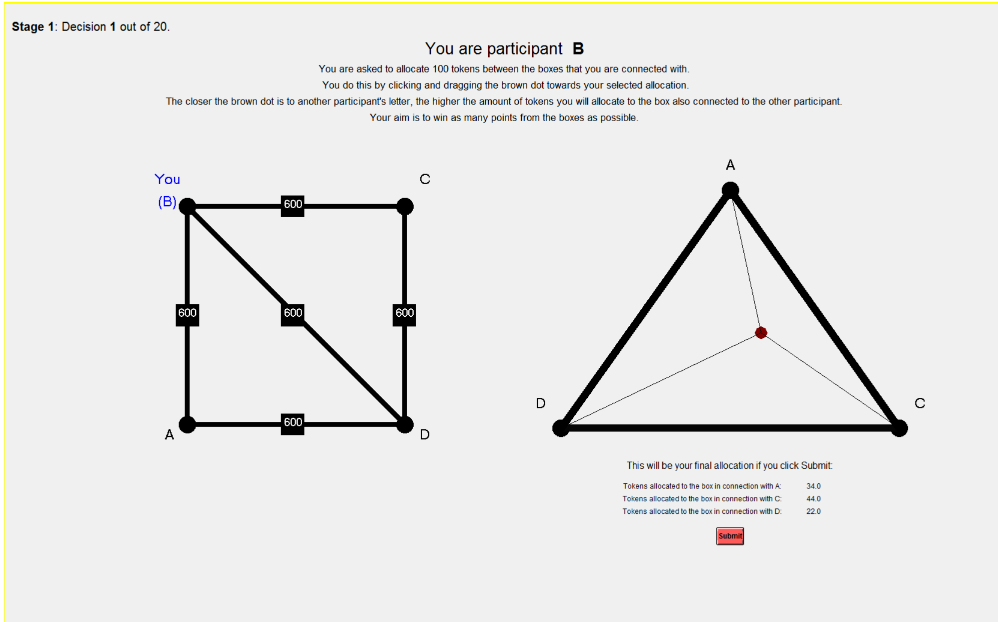



Figure 4: Decision interface for participants with three conflicts

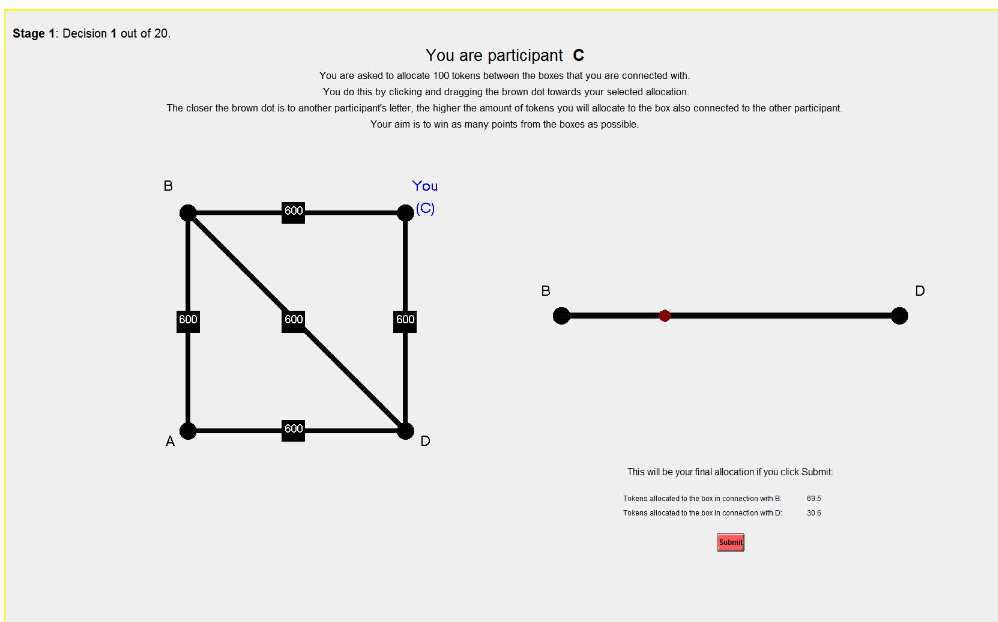



Figure 5: Decision interface for participants with two conflicts

# C Potential Confounds

As it is mention in section 5.1, we compare the distribution of the allocations given their location in the interface when participants were taking their decisions. In Table 5, we present, for each treatment, the vector with the average allocation share to the bottom left, top center and bottom right in the case of the triangle interface and left and right in the case of the slider interface. It is apparent that allocations shares do not depend on the location of the boxes.

Table 5: Average Relative Share across Locations and Test for Order Effects

| | | Homogeneous | Heterogeneous |
|---|---|---|---|
| Ring | | (0.512,0.487) | (0.528,0.471) |
| Complete | | (0.331,0.333,0.336) | (0.345,0.308,0.345) |
| Line | $i = \{2\}$ | (0.509,0.490) | (0.496,0.503) |
| | $i = \{3\}$ | | (0.522,0.477) |
| Diagonal | $i = \{1,3\}$ | (0.507,0.493) | (0.501,0.498) |
| | $i = \{2,4\}$ | (0.339,0.325,0.334) | (0.352,0.316,0.331) |

We compare the relative shares according to their positions in the interface. We present the results of the Friedman test using *** for p<0.01, ** for p<0.05 and * for p<0.1 to show statistical differences.

For all our treatments, the results associated to the Friedman tests do not allow us to reject the null hypothesis that the allocation shares across position are not systematically different.

To test if the representation of the network is not neutral and have behavioural effects, first we present a series of Friedman test, where we compare the average allocation vector of participants allocated to a given position across the four positions in Table 6.

Alternatively, we could compare the distribution of allocation shares for a given box by the assigned position in the each network : position 1, position 2, position 3 or position4. The results of this alternative way to think about representation effects are shown in Table 7.

In total, we conducted 18 Friedman tests: 6 for the Complete network, 2 for the Ring network, 8 for the Diagonal network and 2 for the Line Network. Note that for participants with two battlefields the p-values of comparing the allocations in box1 across position is the same than comparing the allocations in box2 across positions. Using the alternative formulation, we reach the same conclusion about the lack of effects of due to position effects as in the main text.

Table 6: Average Relative Shares across Boxes and Position and Test for Position Effects

| | | Homogeneous | Heterogeneous | |
|---|---|---|---|---|
| Ring | $i = \{1\}$ | (0.483,0.516) | (0.642,0.357) | |
| | $i = \{2\}$ | (0.513,0.486) | (0.653,0.364) | ** |
| | $i = \{3\}$ | (0.524,0.475) | (0.715,0.284) | |
| | $i = \{4\}$ | (0.512,0.488) | (0.774,0.225) | |
| Complete | $i = \{1\}$ | (0.333,0.341,0.326) | (0.510,0.295,0.195) | |
| | $i = \{2\}$ | (0.349,0.324,0.325) | (0.495,0.295,0.209) | ** |
| | $i = \{3\}$ | (0.313,0.330,0.356) | (0.519,0.288,0.191) | |
| | $i = \{4\}$ | (0.340,0.302,0.357) | (0.510,0.3120.177) | |
| Line | $i = \{2\}$ | (0.617,0.382) | – | |
| | $i = \{3\}$ | (0.541,0.468) | – | |
| Diagonal | $i = \{1\}$ | (0.497,0.502) | (0.640,0.359) | |
| | $i = \{3\}$ | (0.477,0.522) | (0.542,0.457) | |
| | $i = \{2\}$ | (0.337,0.306,0.356) | (0.542,0.285,0.171) | |
| | $i = \{4\}$ | (0.333,0.297,0.369) | (0.495,0.309,0.195) | |

Average realtive shares allocated to a given conflict by participants in the same position. Results of the Friedman test at 1%, 5% and 10% significance level are denoted by ***, ** and *, respectively.

Table 7: Test for Order Position associated Friedman Test P-values

| | | Homogeneous | | | Heterogeneous | | |
|---|---|---|---|---|---|---|---|
| | | $a_1$ | $a_2$ | $a_3$ | $a_1$ | $a_2$ | $a_3$ |
| Ring | | 0.8310 | 0.8310 | | 0.3877 | 0.3877 | |
| Complete | | 0.9767 | 0.2508 | 0.9160 | 0.4297 | 0.8081 | 0.8312 |
| Line | $i = \{2\}$ | 0.1721 | 0.1721 | | | — | |
| | $i = \{3\}$ | | | | | — | |
| Diagonal | $i = \{1, 3\}$ | 0.4790 | 0.4790 | | 0.8458 | 0.8458 | |
| | $i = \{2, 4\}$ | 0.2541 | 0.3432 | 0.3709 | 0.3106 | 0.6875 | 0.2689 |

This set of robustness results reinforce the idea that we can abstract position effects and focus on the allocation shares of boxes.

# D Trend analysis

We estimate a Dirichlet Covariate Model to test for time trends. As it is mention in section 2, this analysis relies in the simultaneous estimation of the concentration parameters of a Dirichlet process that minimize the distance between the observed and the predicted data given a set of covariates. For this purpose, we consider the following specification,

$$ln(\alpha_{tk}) = \beta_{0_k} + \beta_{1_k} HT_t + \beta_{2_k} Rounds_t + \beta_{3_k} (Rounds_t \times HT_t) + \boldsymbol{\gamma X}_t + \epsilon_t$$

where $\boldsymbol{X}$ represents a vector of control variables, in this case dummy variables for session and positions. Notice, that the number of simultaneous equations that we need to estimate correspond to the number of battlefields that a participants is facing. Table **??** shows the estimates of the Complete and Diagonal network for participants with degree 3.

Table 8: Dirichlet Covariate Model for Time trends (Degree 3 participants)

| | Complete Network | | | Diagonal Network | | |
|---|---|---|---|---|---|---|
| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ | $\ln(\alpha_3)$ | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ | $\ln(\alpha_3)$ |
| Heterogeneous values | 1.24** | 1.18** | 0.47 | 0.07 | -0.12 | -0.89 |
| | (0.51) | (0.46) | (0.49) | (0.48) | (0.58) | (0.54) |
| Round | -0.01 | -0.01 | -0.01 | 0.01 | 0.03 | 0.02 |
| | (0.01) | (0.01) | (0.02) | (0.03) | (0.02) | (0.02) |
| Round × Heterogeneous | -0.03 | -0.01 | -0.03 | -0.01 | -0.00 | -0.00 |
| | (0.02) | (0.02) | (0.02) | (0.03) | (0.03) | (0.03) |
| Constant | 0.74* | 0.48 | 0.85* | -0.18 | -0.56** | -0.13 |
| | (0.40) | (0.37) | (0.47) | (0.44) | (0.27) | (0.37) |
| Observations | 1,280 | 1,280 | 1,280 | 1,280 | 1,280 | 1,280 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

As we are interested in the expected values of the estimated distribution conditional on our covariates, we will observed a time trend effect if the estimates for each dimension are different from each other. Formally, we will observe a time trend effect if

$$(\beta_{2_1} \neq \beta_{2_2} \neq \beta_{2_3}) \vee (\beta_{3_1} \neq \beta_{3_2} \neq \beta_{3_3})$$

When we conduct the joint hypotheses test for each network using the non-linear Wald test of the condition above we cannot reject the null hypotheses. This suggests that statistically we do not observe time trend effects.

Table 9: Dirichlet Covariate Model for Time Trends (Degree 2 Participants)

| | Diagonal Network | | Ring Network | | Line Network | |
|---|---|---|---|---|---|---|
| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ |
| Heterogeneous values | 0.28 | 0.74** | -0.15 | 0.10 | 0.03 | -0.41 |
| | (0.43) | (0.35) | (0.45) | (0.52) | (0.31) | (0.30) |
| Round | 0.02 | 0.03 | 0.02 | 0.00 | 0.03** | 0.04*** |
| | (0.02) | (0.02) | (0.02) | (0.02) | (0.01) | (0.01) |
| Round × Heterogeneous | -0.03 | -0.04* | -0.03 | -0.00 | -0.01 | -0.02 |
| | (0.02) | (0.02) | (0.02) | (0.03) | (0.01) | (0.01) |
| Constant | 1.83*** | 1.63*** | 0.75** | 0.92*** | 0.29** | 0.33 |
| | (0.44) | (0.48) | (0.30) | (0.28) | (0.14) | (0.23) |
| Observations | 1,280 | 1,280 | 1,280 | 1,280 | 2,160 | 2,160 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

We conduct the same exercise for the Degree 2 participants in the Diagonal, Ring and Line networks. Table 9 shows the estimates of the model. For participants with degree 2, we will observe a time trend effect if

$$(\beta_{2_1} \neq \beta_{2_2}) \vee (\beta_{3_1} \neq \beta_{3_2})$$

We cannot reject the null hypotheses in one out of six tests. That suggests that for participants with degree 2 we do not observe time trend effects.

As robustness analysis, and to discard that our results are driven but the specific error process that we are assuming, we conduct the non-parametric Wilcoxson signed rank sum test to compare the choices made in the first 10 rounds with the choices of the second 10 rounds of the same treatment. If the choices changes systematically with the repetition of the game, we should observe differences at the beginning and the end of each treatment. Table 10 shows the results of this exercise.

The Wilcoxson signed rank sum test presented previously compares the distributions of decisions in the first and second 10 rounds of a given treatment. We could also compare only the means using a binomial test. Table 11 shows the threshold probabilities associated to each hypothesis test.

Note that all the threshold probabilities presented in the Table 11 are lower than 0.3833 and 0.4014 the test reference values. Thus, the null hypotheses, that the average choice in the first 10 rounds of a treatment is equal to the average choice in the second 10 rounds of the same treatment, cannot be rejected. Using both non-parametric methods, we do not find systematic difference between the beginning or end of a treatment decisions. For our design the results of the parametric and

Table 10: Non Parametric Analysis of Time Trends per Network and Component and associated P-Values

| | | Homogeneous | | | Heterogeneous | | |
|---|---|---|---|---|---|---|---|
| | | $a_1$ | $a_2$ | $a_3$ | $a_1$ | $a_2$ | $a_3$ |
| Ring | | 0.3771 | 0.3771 | | 0.3771 | 0.3771 | |
| Complete | | 0.8601 | 0.0501 | 0.3771 | 0.3771 | 0.1102 | 0.5966 |
| Line | $i = \{2\}$ | 0.2478 | 0.2478 | | 1.0000 | 1.0000 | |
| | $i = \{3\}$ | 0.4421 | 0.4421 | | 0.7011 | 0.7011 | |
| Diagonal | $i = \{1, 3\}$ | 0.0201 | 0.0201 | | 1.0000 | 1.0000 | |
| | $i = \{2, 4\}$ | 0.5966 | 0.2153 | 1.0000 | 0.8601 | 0.1102 | 1.0000 |

To control the potential effects over inference due to multiple hypothesis tests, we applied the Bonnferonni correction where we test each individual hypothesis at confidence interval of $1 - \alpha = \frac{0.05}{20}$.

Table 11: Threshold probability of Binomial Tests for Time Trends per Network and Component.

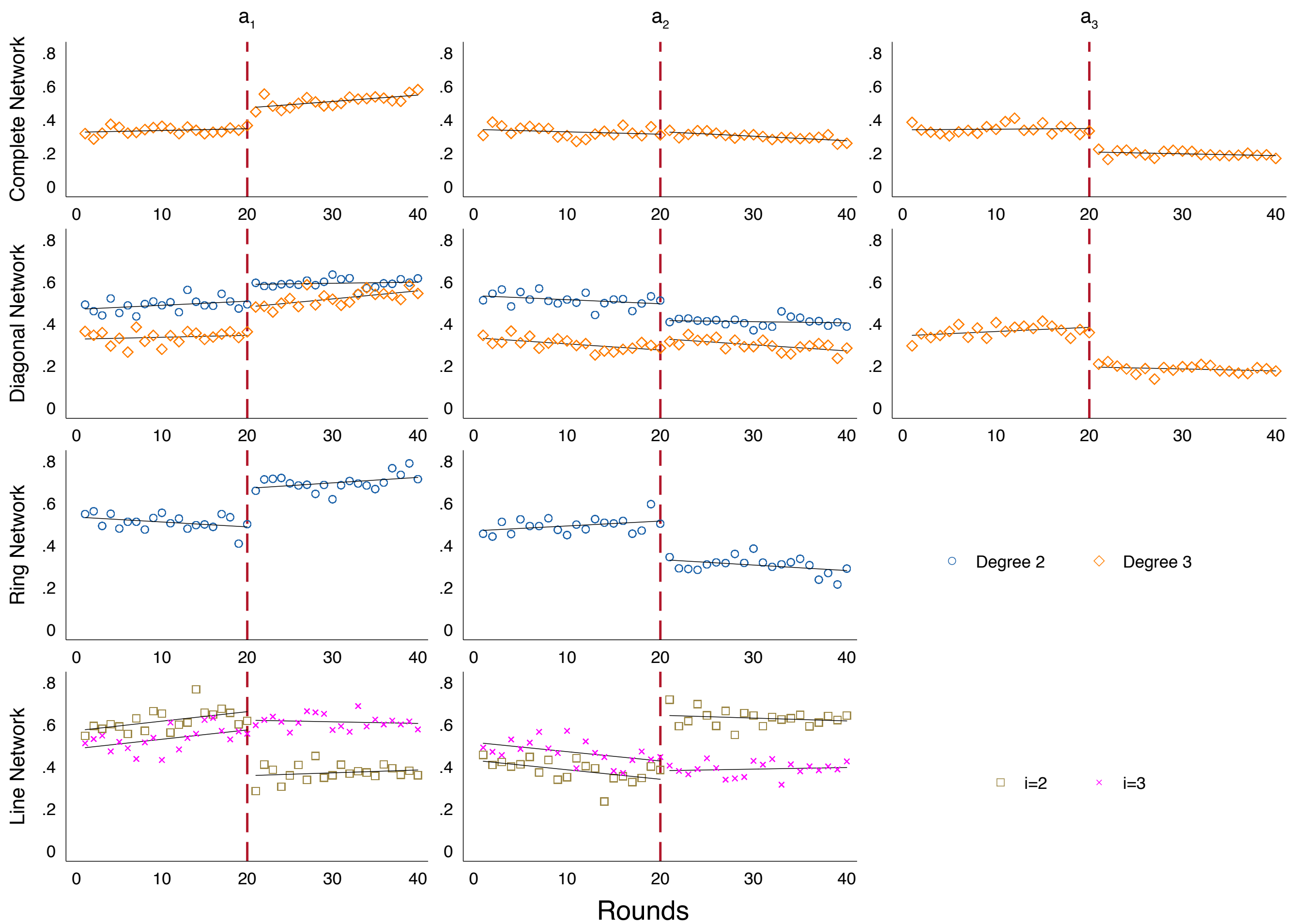
| | | Homogeneous | | | Heterogeneous | | |
|---|---|---|---|---|---|---|---|
| | | $a_1$ | $a_2$ | $a_3$ | $a_1$ | $a_2$ | $a_3$ |
| Ring | | 0.0196 | 0.0196 | | 0.0201 | 0.0201 | |
| Complete | | 0.0184 | 0.0230 | 0.0215 | 0.0225 | 0.0227 | 0.0191 |
| Line | $i = \{2\}$ | 0.0203 | 0.0203 | | 0.0137 | 0.0137 | |
| | $i = \{3\}$ | 0.0331 | 0.0331 | | 0.0151 | 0.0151 | |
| Diagonal | $i = \{1, 3\}$ | 0.0192 | 0.0192 | | 0.0217 | 0.0217 | |
| | $i = \{2, 4\}$ | 0.0199 | 0.0232 | 0.0226 | 0.0201 | 0.0205 | 0.0227 |

non-parametric test implies that we do not observe learning effects in our environment.

To test this formally, we estimate a Dirichlet Covariate model controlling for rounds, a stage dummy and the interaction effect considering position and session fix effects (see details in Appendix D). The results of the test show that there are no learning effects (in which the allocation share from one box is adjusted to increase or decrease the share of a different box as rounds progress). We also find that dispersion of allocation shares increases in the second stage of two networks (complete and diagonal) but decreases in the first stage of another network (line). In addition, we also run a robustness check at the individual level, we estimate the same model but now for each participants separately and we test if a given participants exhibit learning effects. The results show that 63% of our sample do not display behavioral changes across rounds.

Another concern in our experiment is the possibility of learning with the feedback received between rounds. Figure 6 presents the average allocation share to each conflict by round for each network and type of participant. A dashed vertical red line separates the end of the first stage and the beginning of the second stage. With a casual look, we observe small time trends within each stage, if any.

Figure 6: Average Relative Shares over Rounds



# E Equilibrium Test

## E.1 Estimated parameters of Dirichlet Covariate Model per networks

To test our main set of hypotheses, in a first instance we need to estimate the Dirichlet Covariate Model for each network and type of participants – degree 3 or degree 2. We would like to estimate simultaneously the concentration parameters of a Dirichlet process that minimize the distance between the observed and predicted data given a set of covariates. As we do not observe learning effects, see appendix D, to capture the idiosyncratic characteristics of each participant, our unit of analysis is going to be the participant's average allocation share per treatment per conflict. We consider the following econometric specification,

$$\ln(\alpha_k) = \beta_{0_k} + \beta_{1_k} HT_i + \boldsymbol{\gamma X}_i$$

where $HT_t$ is a dichotomous variable equal to one when the battlefield values are heterogeneous and zero otherwise. As controls we include fix effects of sessions and we cluster the standard errors by group of participants interacting with each other. Tables 12, 13, 14, 15 and 16 show the estimates for the participants in the complete network, ring network, diagonal network with degree 2, diagonal network with degree 3 and line network, respectively.

Table 12: Dirichlet Covariate Model for the Complete Network

| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ | $\ln(\alpha_3)$ |
|---|---|---|---|
| Heterogeneous values | -1.04*** | -1.60*** | -2.24*** |
| | (0.39) | (0.38) | (0.41) |
| Constant | 3.09*** | 3.17*** | 3.22*** |
| | (0.37) | (0.32) | (0.3) |
| Observations | 64 | 64 | 64 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

For the Complete network, we conduct the following hypotheses test based on our estimates: $\exp(\hat{\beta}_{0_1}) = \exp(\hat{\beta}_{0_2}) = \exp(\hat{\beta}_{0_3})$ and $\frac{2}{3}\exp(\hat{\beta}_{0_1} + \hat{\beta}_{1_1}) = \exp(\hat{\beta}_{0_2} + \hat{\beta}_{1_2}) = 2\exp(\hat{\beta}_{0_3} + \hat{\beta}_{1_3})$ for the Homogeneous and Heterogenous treatments, respectively.

Table 13: Dirichlet Covariate Model for the Ring Network

| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ |
|---|---|---|
| Heterogeneous values | -2.49*** | -3.24*** |
| | (0.80) | (0.86) |
| Constant | 5.27*** | 5.27*** |
| | (0.66) | (0.66) |
| Observations | 64 | 64 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

For the Ring network, we conduct the following hypotheses test based on our estimates: $\exp(\hat{\beta}_{0_1}) = \exp(\hat{\beta}_{0_2})$ and $\frac{1}{3}\exp(\hat{\beta}_{0_1} + \hat{\beta}_{1_1}) = \exp(\hat{\beta}_{0_2} + \hat{\beta}_{1_2})$ for the Homogeneous and Heterogenous treatments, respectively.

Table 14: Dirichlet Covariate Model for participants with degree two in the Diagonal network.

| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ |
|---|---|---|
| Heterogeneous values | -1.61*** | -2.07*** |
| | (0.69) | (0.97) |
| Constant | 4.33*** | 4.29*** |
| | (0.80) | (0.76) |
| Observations | 64 | 64 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

For participants in the diagonal network with Degree two, we conduct the following hypotheses test based on our estimates: $\exp(\hat{\beta}_{0_1}) = \exp(\hat{\beta}_{0_2})$ and $\frac{2}{3}\exp(\hat{\beta}_{0_1} + \hat{\beta}_{1_1}) = \exp(\hat{\beta}_{0_2} + \hat{\beta}_{1_2})$ for the Homogeneous and Heterogenous treatments, respectively.

Table 15: Dirichlet Covariate Model for participants with degree three in the Diagonal network

| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ | $\ln(\alpha_3)$ |
|---|---|---|---|
| Heterogeneous values | -2.29** | -2.65*** | -3.03*** |
| | (0.52) | (0.53) | (0.50) |
| Constant | 3.22*** | 3.58*** | 3.64*** |
| | (0.32) | (0.36) | (0.34) |
| Observations | 64 | 64 | 64 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

For participants in the diagonal network with Degree 3, we conduct the following hypotheses test based on our estimates: $\exp(\hat{\beta}_{0_1}) = \exp(\hat{\beta}_{0_2}) = \frac{0.328}{0.343}\exp(\hat{\beta}_{0_3})$ and $\frac{0.332}{0.499}\exp(\hat{\beta}_{0_1} + \hat{\beta}_{1_1}) = \exp(\hat{\beta}_{0_2} + \hat{\beta}_{1_2}) = \frac{0.332}{0.167}\exp(\hat{\beta}_{0_3} + \hat{\beta}_{1_3})$ for the Homogeneous and Heterogenous treatments, respectively.

For the line network, we need to consider that participants assigned positions 2 and 3 are facing a different set of battlefield values in the heterogeneous values treatment. Therefore, we need to take into account the differential effects of being assigned to position 2 or 3 and the heterogeneous effect of the treatment given the assigned position. In particular, we consider the following specification,

$$\ln(\alpha_k) = \beta_{0_k} + \beta_{1_k} HT_i + \beta_{2_k} Position2_i + \beta_{3_k} HT_i \times Position2_i + \boldsymbol{\gamma}\boldsymbol{X}_i$$

where the variable $Position2_i$ is equal to one if the participant $i$ was assigned to the position 2 and zero otherwise. In the Homogeneous treatment, we conduct the following hypothesis test based on our estimates $\frac{0.465}{0.535}\exp(\hat{\beta}_{0_1}) = \exp(\hat{\beta}_{0_2})$. In the Heterogeneous treatment, the values of the conflicts induces different predictions for participants in positions 2 and position 3. In these case, we perform two hypotheses tests: $\exp(\hat{\beta}_{0_1} + \hat{\beta}_{1_1} + \hat{\beta}_{2_1} + \hat{\beta}_{3_1}) = \frac{0.328}{0.672}\exp(\hat{\beta}_{0_2} + \hat{\beta}_{1_2} + \hat{\beta}_{2_2} + \hat{\beta}_{3_2})$ and $\exp(\hat{\beta}_{0_1} + \hat{\beta}_{1_1}) = \frac{0.457}{0.543}\exp(\hat{\beta}_{0_2} + \hat{\beta}_{1_2})$

Table 16: Dirichlet Covariate Model for the Line network.

| | $\ln(\alpha_1)$ | $\ln(\alpha_2)$ |
|---|---|---|
| Heterogeneous values | -1.64*** | -1.65*** |
| | (0.01) | (0.02) |
| Position 2 | -1.71*** | -1.76*** |
| | (0.01) | (0.01) |
| Heterogeneous values × Position 2 | 0.856*** | 1.25** |
| | (0.01) | (0.01) |
| Constant | 5.00*** | 4.85*** |
| | (0.35) | (0.31) |
| Observations | 108 | 108 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notice that all our hypothesis are made in term of the concentration parameters of the estimated distributions while our estimations are done in terms of the natural logarithms of these parameters. Even thought there is mathematically equivalence between $\beta_{0_{box1}} = \beta_{1_{box1}}$ and $\exp(\beta_{0_{box1}}) = \exp(\beta_{1_{box1}})$, from an statistical perspective the standard errors of $\beta_{0_{box1}}$ and $\exp(\beta_{0_{box1}})$ are not the same. To control by this lag invariance of the representation, we use the Wald test for nonlinear hypotheses in which the standard errors and p-values are adjusted using the Delta method.

## E.2 Robustness analysis

Table 17: Expected Values of the OLS Estimates

| | | Homogeneous | Heterogeneous |
|---|---|---|---|
| Ring | | (0.504,0.495) | (0.316,0.683)*** |
| Complete | | (0.332,0.262,0.404)*** | (0.305,0.437,0.256)*** |
| Line | p=2 | (0.473,0.526) | (0.635,0.364) |
| | p=3 | | (0.393,0.606) |
| Diagonal | d=2 | (0.534,0.465)*** | (0.430,0.569)*** |
| | d=3 | (0.296,0.294,0.408)*** | (0.292,0.478,0.229)*** |

For participants with degree 3, the first element of the vector each correspond to $a_1^*$, the second element to $a_2^*$ and the third element of $a_3^*$. Similarly, for participants with degree 2 the first element of each vector correspond to $a_1^*$ and the second element to $a_2^*$. We present the results of the non-linear Wald test using *** for p<0.01, ** for p<0.05 and * for p<0.1.

Table 18: Percentage of Participants for which we reject the Nash equilibrium Behaviour using OLS estimates

| | | Homogeneous | Heterogeneous |
|---|---|---|---|
| Ring | | 0% | 100% |
| Complete | | 0% | 100% |
| Line | p=2 | 100% | 0% |
| | p=3 | | 100% |
| Diagonal | d=2 | 0% | 0% |
| | d=3 | 100% | 100% |

# F Sources of Information and Deviations from Equilibrium

## F.1 Main Analysis

To investigate the deviations from equilibrium, we compute the euclidean distance from the observed choices and the equilibrium choices. We want to see if been in a more complex environments induces higher deviations. To do so, we conduct a regression analysis controlling by participants random effects, fix effects of session and clustering the standard errors by group of participants interacting with each other. Table 19 shows the estimates.

Table 19: Determinants of the Deviations from Equilibrium

| | Euclidian distance to Equilibrium actions | Euclidian distance to Equilibrium payoffs |
|---|---|---|
| HT | 0.023 | 29.260** |
| | (0.018) | (14.153) |
| D=3 | 0.081*** | 35.800*** |
| | (0.021) | (8.095) |
| Diverse | 0.077*** | -9.701 |
| | (0.020) | (8.793) |
| HT×D=3 | -0.002 | -10.320 |
| | (0.020) | (12.800) |
| HT×Diverse | -0.008 | -9.454 |
| | (0.014) | (10.686) |
| D=3×Diverse | -0.076*** | -2.166 |
| | (0.026) | (12.450) |
| HT×D=3×Diverse | 0.029 | 7.555 |
| | (0.028) | (19.445) |
| Constant | 0.140*** | 88.271*** |
| | (0.019) | (9.664) |
| Observations | 7,280 | 7,280 |
| Number of participants | 182 | 182 |
| Number of rounds per participant | 40 | 40 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

### F.2 Robustness Analysis

In order to check if our specifications are driving the results, we consider some robustness exercises. We present results over the determinants of the deviations from equilibrium using a simple OLS specification.

Table 20: Determinants of the Deviations from Equilibrium OLS

| | Euclidian distance to Equilibrium actions | Euclidian distance to Equilibrium payoffs |
|---|---|---|
| HT | 0.023 | 29.286** |
| | (0.018) | (14.151) |
| D=3 | 0.081*** | 35.796*** |
| | (0.021) | (8.095) |
| Diverse | 0.077*** | -9.691 |
| | (0.020) | (8.794) |
| HT×D=3 | -0.002 | -10.320 |
| | (0.020) | (12.800) |
| HT×Diverse | -0.008 | -9.517 |
| | (0.014) | (10.689) |
| D=3×Diverse | -0.076*** | -2.168 |
| | (0.026) | (12.450) |
| HT×D=3×Diverse | 0.029 | 7.618 |
| | (0.028) | (19.446) |
| Constant | 0.140*** | 88.274*** |
| | (0.019) | (9.664) |
| Observations | 7,280 | 7,280 |
| R-squared | 0.127 | 0.100 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notice that if we compare these results with the regression analysis considering random effects, some of the estimates change slightly but they remain in the same order of magnitude. These estimates also keep the same level of significance than the ones estimated with random effect of participants.

# G Other Decision Rules

We can replicate our analysis by considering different benchmark behaviour that operates with less amount of information. For example, participants taking into account just their degree will select strategies in proportion to their degree ($1/d$ *decision rule*). If, in addition, participants incorporate the weights of the links into their decision process, then, they will choose their strategies in proportion to these weights (*proportional decision rule*). Formally, The proportional rule suggests that the strategy of player $i$ in link $j$ is determined by the relative weight of link $j$, $w(l_{ij})$, with respect to every other link of player $i$ meaning that $a^*_{ij} = \frac{w(l_{ij})}{\sum_j w(l_{ij})}$.

## G.1 Analysis with the $\frac{1}{d}$ Rule as Benchmark

Table 21: Determinants of the Deviations from $\frac{1}{d}$ rule

| | Euclidian distance to $\frac{1}{d}$ actions | Euclidian distance to $\frac{1}{d}$ payoffs |
|---|---|---|
| HT | 0.063*** | 34.122** |
| | (0.019) | (13.450) |
| D=3 | 0.080*** | 37.277*** |
| | (0.021) | (8.300) |
| Diverse | 0.081*** | -7.904 |
| | (0.019) | (8.773) |
| HT×D=3 | 0.014 | -19.200 |
| | (0.031) | (13.174) |
| HT×Diverse | -0.045*** | -22.722** |
| | (0.015) | (11.017) |
| D=3×Diverse | -0.077*** | -6.091 |
| | (0.027) | (12.356) |
| HT×D=3×Diverse | 0.073** | 24.140 |
| | (0.037) | (19.649) |
| Constant | 0.105*** | 82.954*** |
| | (0.016) | (8.694) |
| Observations | 7,280 | 7,280 |
| Number of participants | 182 | 182 |
| Number of rounds per participant | 40 | 40 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

## G.2 Analysis with the Proportional Rule as Benchmark

Table 22: Determinants of the Deviations from Proportional rule

| | Euclidian distance to Proportional actions | Euclidian distance to Proportional payoffs |
|---|---|---|
| HT | 0.020 | 29.487** |
| | (0.018) | (14.164) |
| D=3 | 0.081*** | 35.713*** |
| | (0.021) | (8.118) |
| Diverse | 0.099*** | -5.904 |
| | (0.019) | (8.806) |
| HT×D=3 | -0.002 | -10.299 |
| | (0.020) | (12.777) |
| HT×Diverse | -0.028* | -11.542 |
| | (0.015) | (10.660) |
| D=3×Diverse | -0.097*** | -4.964 |
| | (0.025) | (12.390) |
| HT×D=3×Diverse | 0.049* | 9.007 |
| | (0.029) | (19.388) |
| Constant | 0.142*** | 88.135*** |
| | (0.019) | (9.885) |
| Observations | 7,280 | 7,280 |
| Number of participants | 182 | 182 |
| Number of rounds per participant | 40 | 40 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1